9:10

Wednesday, September 28, 2016

Review

Design Factor / Factor of Safety

$$n_d = \frac{\text{loss of function parameter}}{\text{max allowable parameter}}$$

typically $\frac{\text{strength}}{\text{max stress.}}$ $\frac{S}{\sigma \text{ or } \tau}$

Design Factor: target spec for FoS

Factor of Safety: found after detailed analysis or testing

Example: Factor of Safety Calculation

A square cross section rod is loaded axially with a static load of 1000+/-10 lbs. The strength of the material is 25 kpsi and the desired design factor is 4. Determine the minimum width of the square cross section. Then select a preferred fractional inch size from Table A-17 and report the factor of safety.

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{P_{max}}{w^2}$$

Design Factor: $n_d = \frac{S}{\sigma_{max}} \Rightarrow \sigma_{max} = \frac{S}{n_d}$

$$\sqrt{w^2} = \sqrt{\frac{P_{max} n_d}{S}} \qquad w = \sqrt{\frac{P_{max} n_d}{S}}$$

$$w = \sqrt{\frac{(1010 \text{ lbs})(4)}{25E3 \text{ psi}}} = 0.401995''$$

$$\frac{7}{16}'' = 0.4375''$$ {From Table A-17 "preffered sizes"

$w_p = 0.4375''$

Actual FoS

$$n = \frac{S w_p^2}{P_{max}} = 4.74$$

Selecting Design Factor

- inherently subjective
- follow industry standards

Standard: set of specs to achieve uniformity, efficiency, and quality

Code: specs to control for safety and performance

Selection depends:

- degree of uncertainty about the loading
- degree of uncertainity about the material strength
- consequences of failure (human safety, economics)
- cost of providing a high FoS

FoS calcs are based off absolutes
probability of failure is more realistic

deterministic vs probability

What is wrong with this statement?

"The yield strength of rolled mild steel is 220 MPa."

220 MPa ± 10 MPa
standard dev

Model Uncertainty

reliability, R — statistical measure
probability that something won't fail

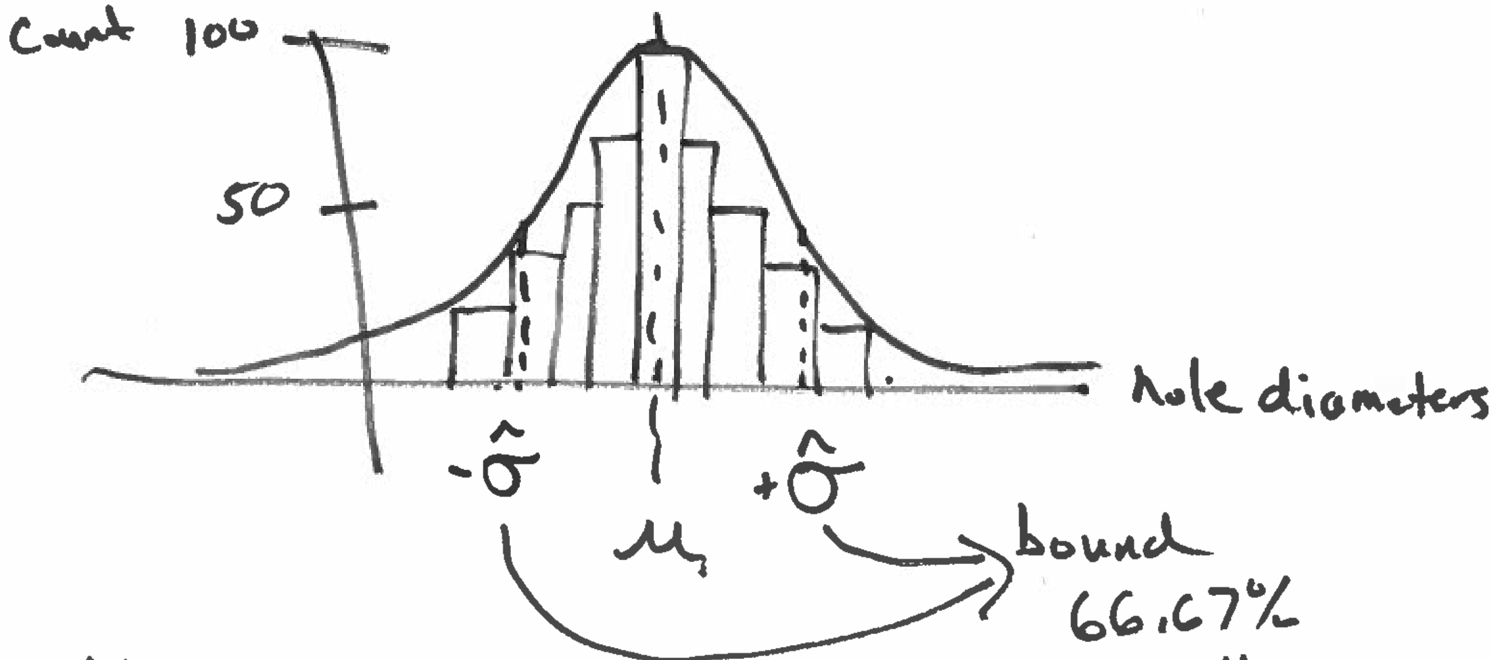
probability of failure, P_f
probability that something will fail

$$1 - R = P_f$$

Probabilities modeled by various distribution funcs.

Most common: Gaussian Normal

Example



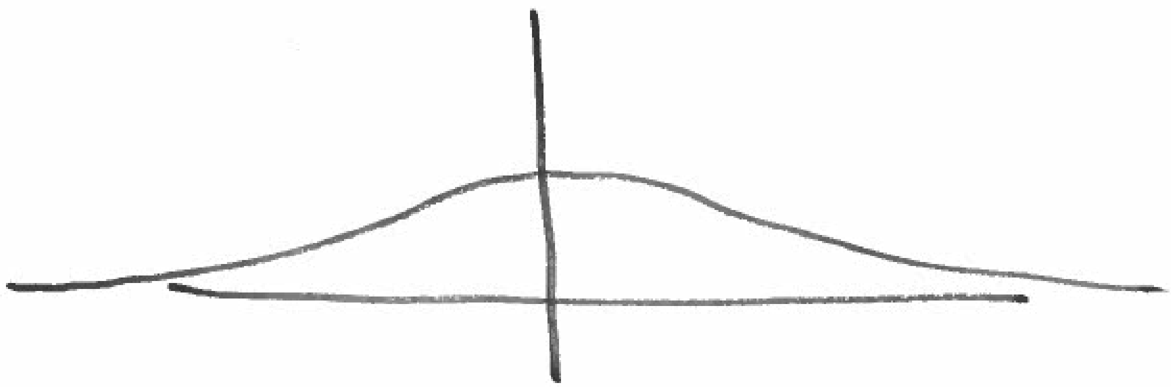
μ: mean

$\hat{\sigma}$: standard deviation

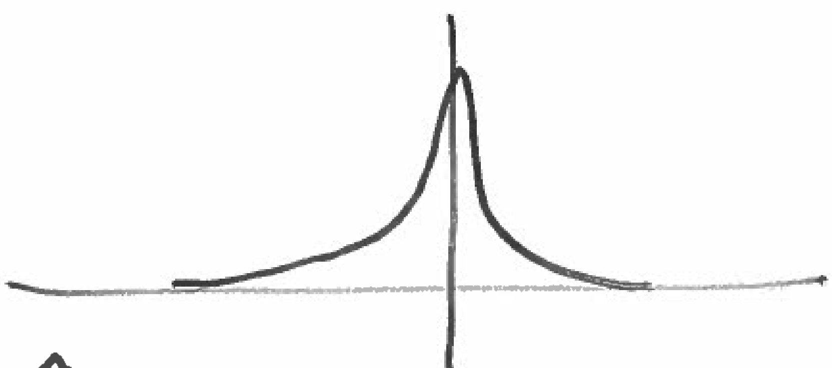
$$f(x) = \frac{1}{\hat{\sigma}\sqrt{2\pi}} e^{\left[-\frac{1}{2}\left(\frac{x-\mu}{\hat{\sigma}}\right)^2\right]}$$

Gaussian Probability Density Function

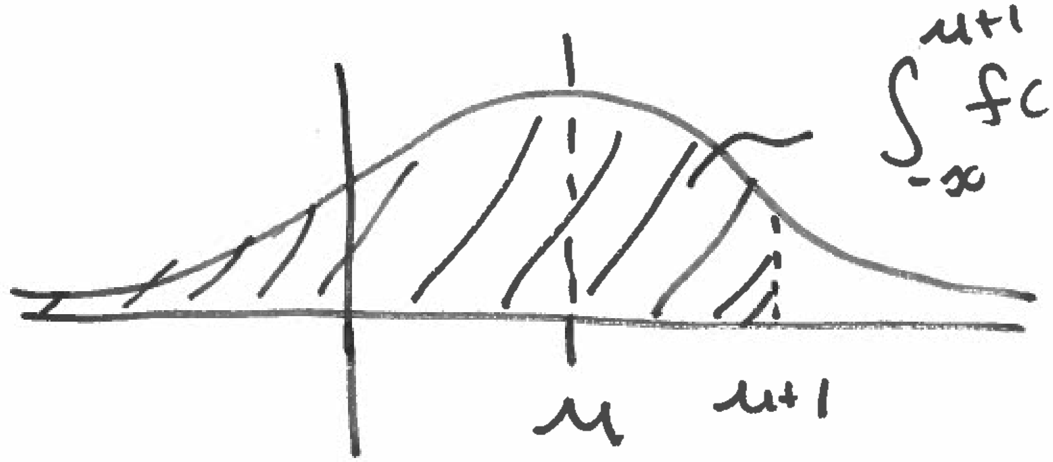
$$\int_{-\infty}^{\infty} f(x) \Rightarrow 100\%$$



$\hat{\sigma}$ is large: more spread



$\hat{\sigma}$ is small



$$\int_{-\infty}^{\mu+1} f(x) = \Phi_{\mu+1}$$

What is the probability that the observation is less than $\mu+1$?

$$Z = \frac{x - \mu}{\hat{\sigma}}$$

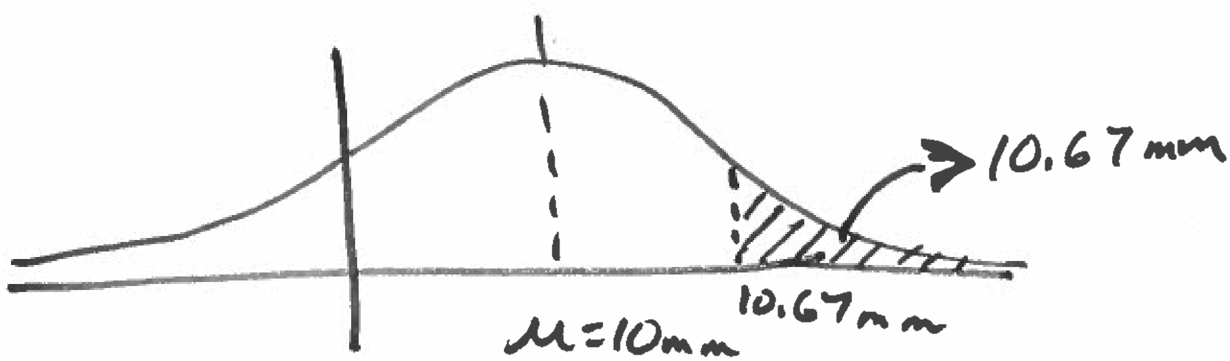
Example

You have 1M parts (hole diameter) and you measure 1000 of them to build a histogram.

$\mu_d = 10$ mm

$\hat{\sigma}_d = 0.5$ mm

What is the probability that a randomly chosen part has a diameter > 10.67 mm?



$$Z = \frac{d - \mu_d}{\hat{\sigma}_d} = \frac{10.67\text{ mm} - 10\text{ mm}}{0.5\text{ mm}} = 1.34$$

↳ find α in Table A-10

270

$\alpha = 0.0901 \approx$ **9%** $\simeq$ 90k