Tolerances & Dimensioning

- Nominal sizes are not exact size
  "2x4"
- tolerances are absolute limits
- tight tolerances = high costs
- dimensioning ⇒ minimal set of dimensions
  no-redundancy
- dimensions should be based on part functionality
- tolerance stack up

Units

S.I. and U.S. Customary Units

Kg m s lb. in s
ft
slug

Mar's climate orbiter!
$600 million loss due to units mistake.

Free Body Diagrams

System

- any isolated part or portion of a machine or structure
- used identify and isolate both internal and externals loads on a design element
- define coordinate system(s)
- define unknowns and knowns

$$\sum \bar{F} = \frac{d\bar{p}}{dt}$$ — linear momentum

Newton's second law

$$\sum \bar{M} = \frac{d\bar{H}}{dt}$$ — angular momentum

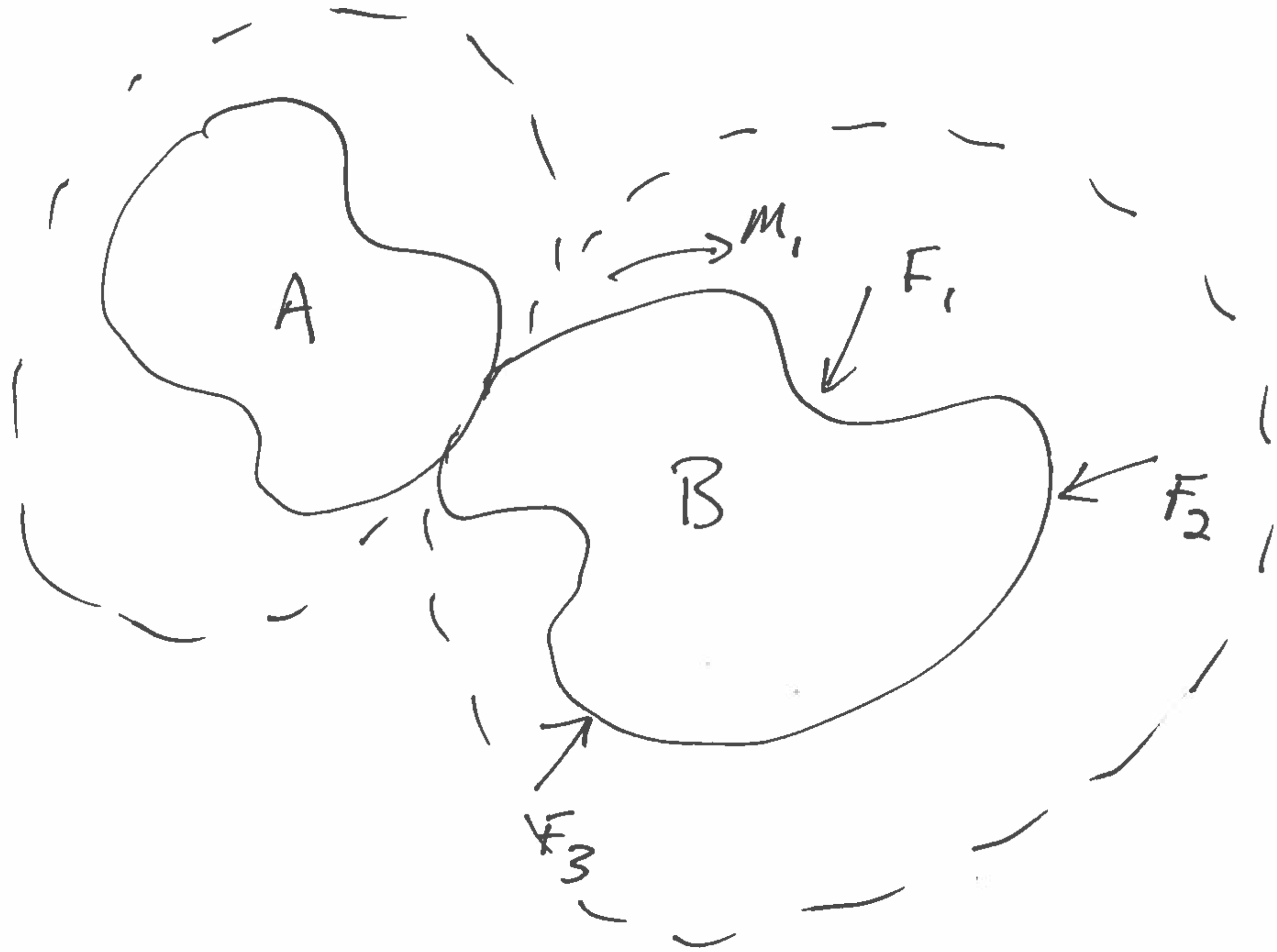
Euler's equation of motion

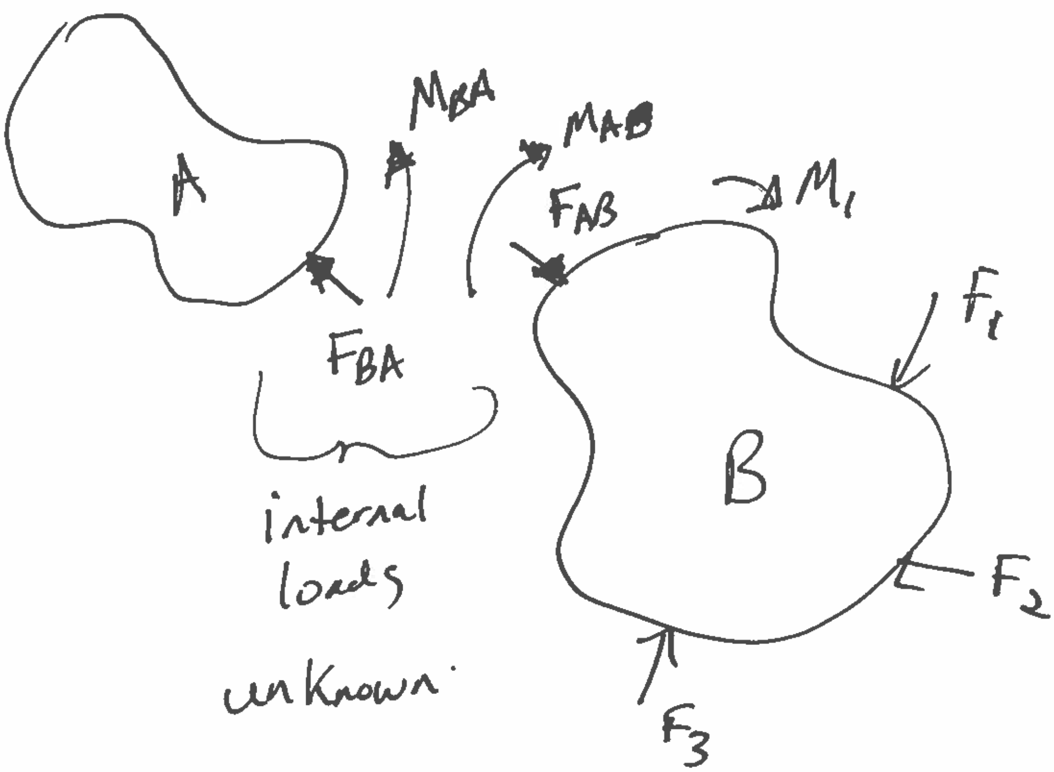
Equilibrium state

$$\frac{d\bar{p}}{dt} = 0 = \frac{d\bar{H}}{dt}$$

$$\boxed{\begin{array}{c}\sum \bar{F} = 0 \\ \sum \bar{M} = 0\end{array}}$$

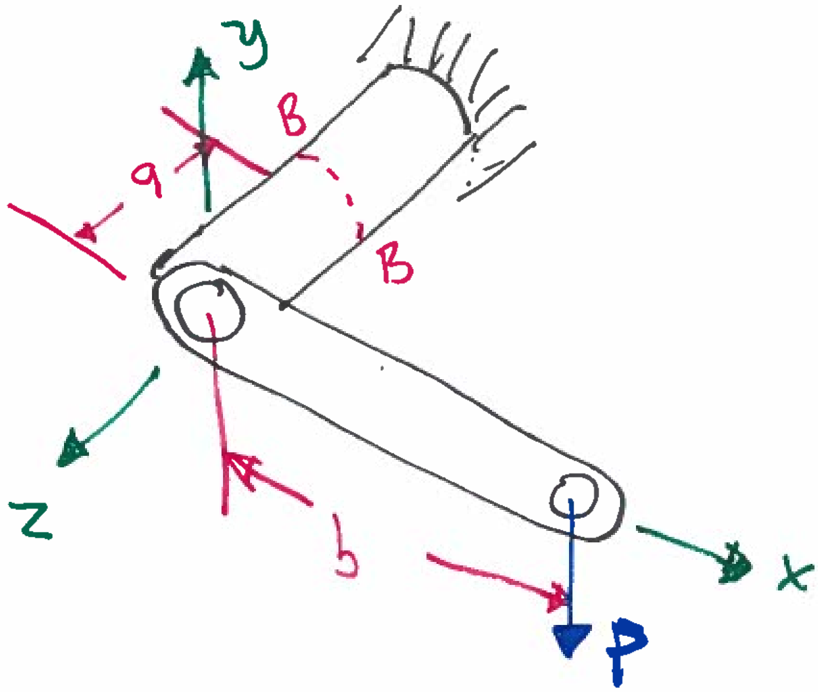
object is at rest

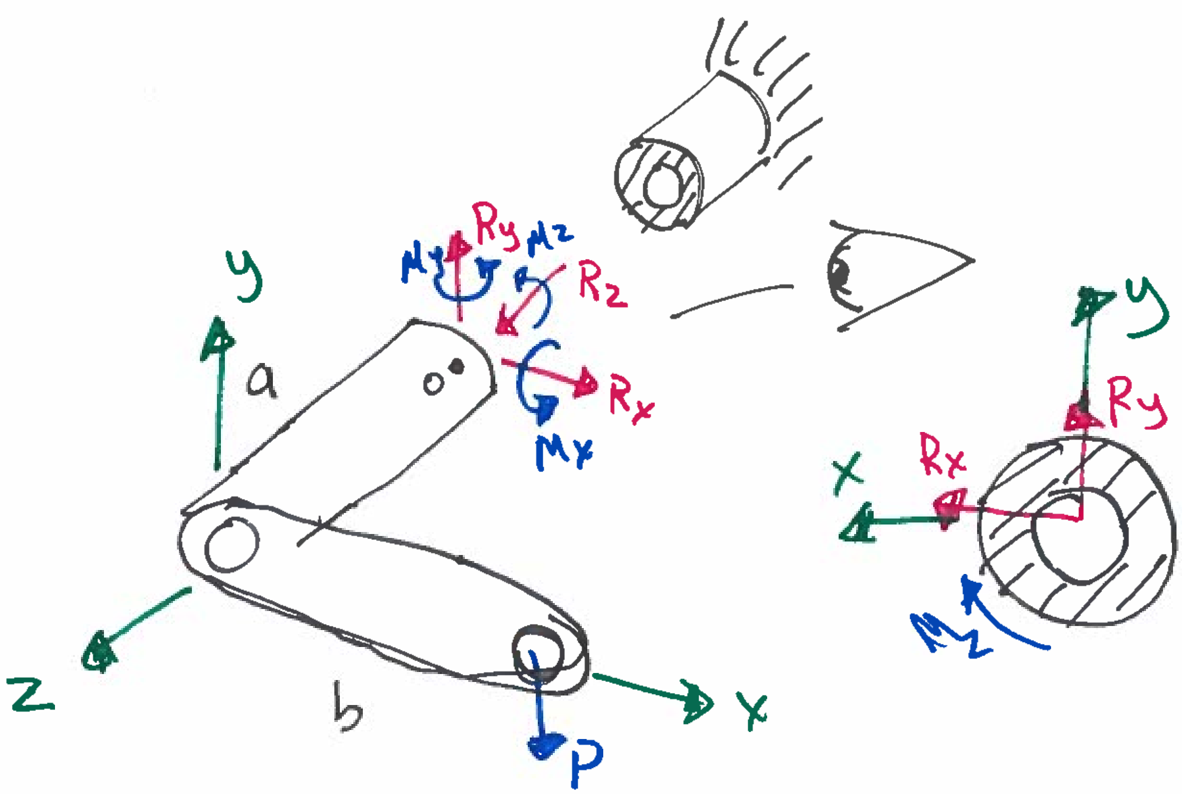

A
B
M1
F1
F2
F3


A
MBA
MAB
FAB
M1
F1
FBA
B
internal
loads
unknown.
F2
F3

Example

Find the internal loads at B-B





$$\underline{\sum F = 0}$$

$$\sum F_x = \boxed{R_x = 0}$$

$$\sum F_y = R_y - P = 0$$

$$\boxed{R_y = P}$$

$$\sum F_z = \boxed{R_z = 0}$$

$$\underline{\sum M_o = 0}$$

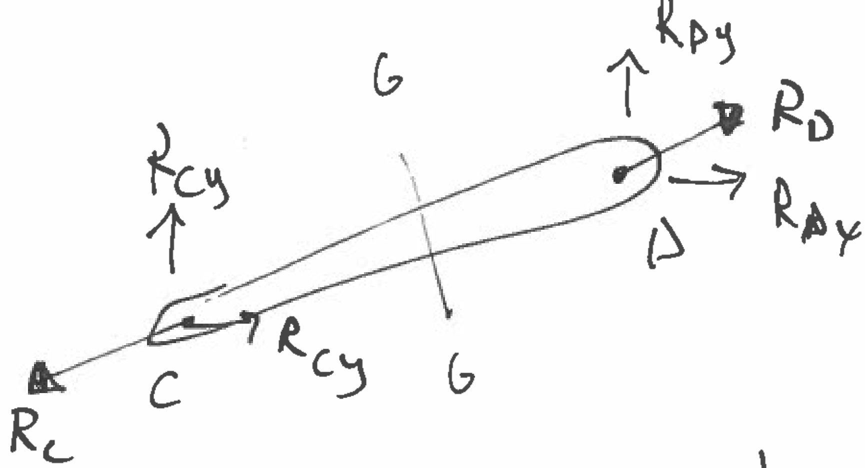
$$\sum M_x^o = M_x + Pa = 0$$

$$\boxed{M_x = -Pa}$$

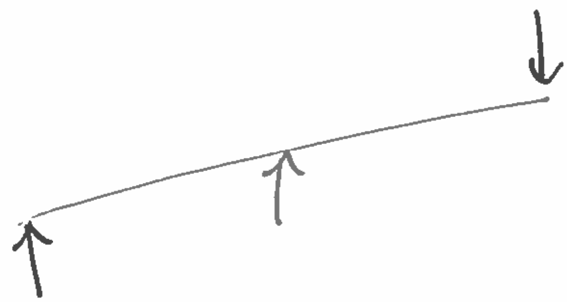
$$\sum M_y^o = \boxed{M_y = 0}$$

$$\sum M_z^o = M_z - Pb = 0$$

$$\boxed{M_z = Pb}$$



$R_C = -R_D$

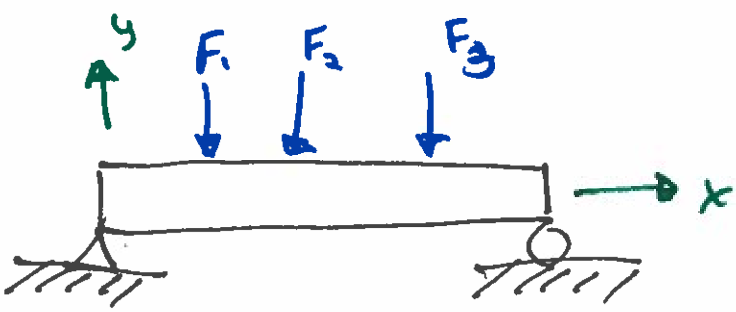


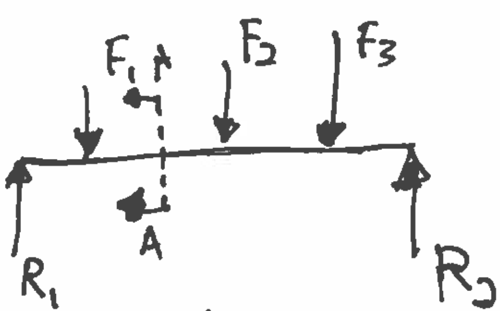
Transverse Loading of Slender Beams

Beam: slender element with

$$\frac{\text{length}}{\text{width}} \geq 10$$

The beam supports transverse loads

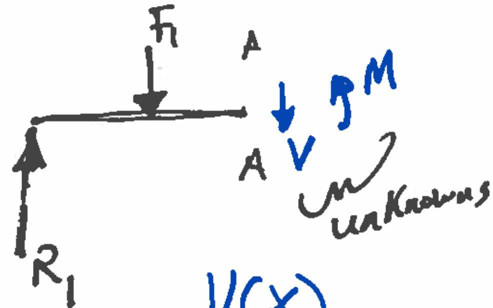




FBD

$\Rightarrow$

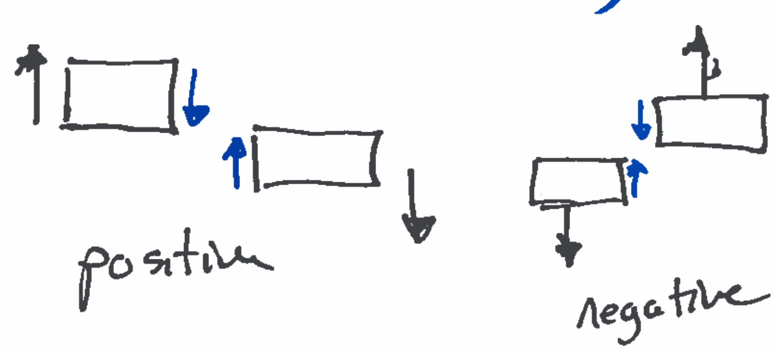
Method of sections



$V(x)$

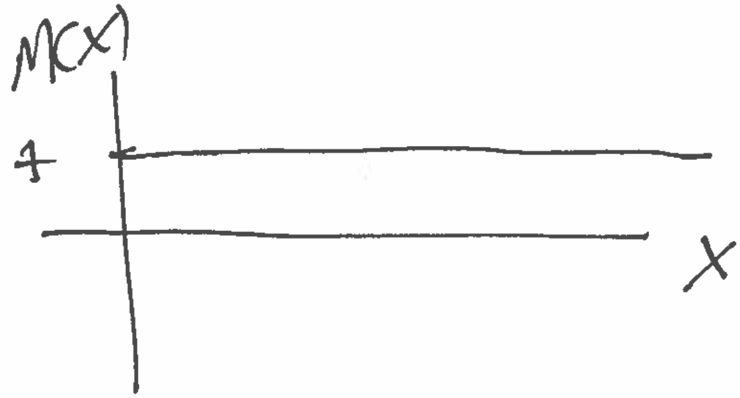
$M(x)$

Shear



Bending

positive   neg.

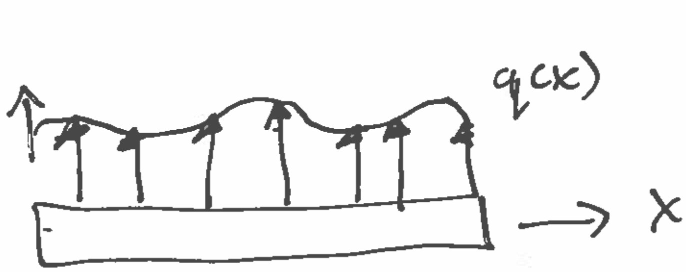


For equilibrium at any transverse section the shear reaction, $V(x)$, and the bending moment, $M(x)$, will be present.

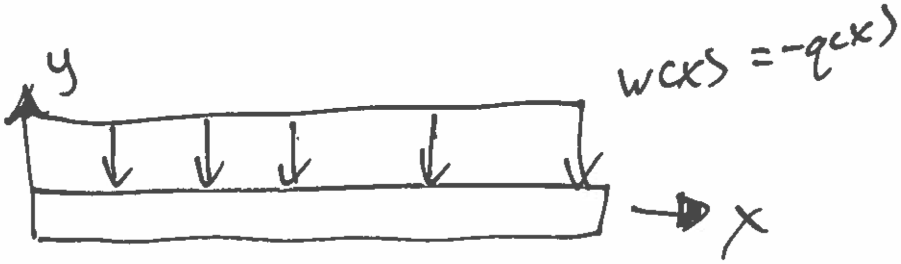
$$\boxed{V(x) = \frac{d\,M(x)}{dx}}$$

The load on the beam, $q(x)$, is related by:

$$\boxed{q(x) = \frac{dV(x)}{d(x)} = \frac{d^2 M(x)}{dx^2}}$$



q(x): any distributed load
and also concentrated loads



If q(x) is known:

$$\int dV(x) = \int q(x)\,dx \Rightarrow \underbrace{V(x)}_{\text{finding}} = \underbrace{\int q(x)\,dx}_{\text{given}}$$

$$M(x) = \int V(x)\,dx$$

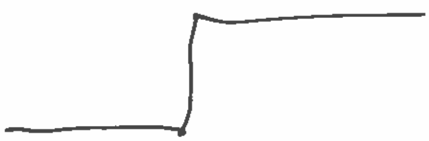
each integration has constant of integration
found by the boundary conditions

Singularity Functions

- allow us to use a single expression for whole beam (instead of piecewise)

- these functions: - dirac delta functions
  - unit doublet functions
  - Heaviside functions

dirac delta



Heaviside

Distributed Loads

$n \geq 0$

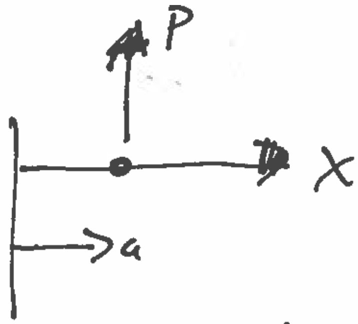
$$\langle x-a \rangle^n = \begin{cases} 0 & x < a \\ (x-a)^n & x \geq a \end{cases}$$

coordinate along beam

location of the discontinuity

$$\int \langle x-a \rangle^n \, dx = \frac{\langle x-a \rangle^{n+1}}{n+1} + C$$

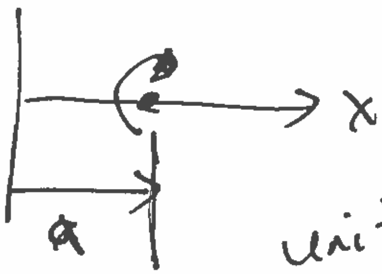
Concentrated Loads



$$q(x) = P\langle x-a\rangle^{-1}$$

$$= \begin{cases} 0 & \text{for } x \neq a \\ +\infty & \text{for } x = a \end{cases}$$

dirac delta



$$q(x) = M\langle x-a\rangle^{-2}$$

$$= \begin{cases} 0 & x \neq a \\ \pm\infty & x = a \end{cases}$$

unit doublet

$n<0$

$$\int \langle x-a\rangle^{n}\,dx = \langle x-a\rangle^{n+1}$$