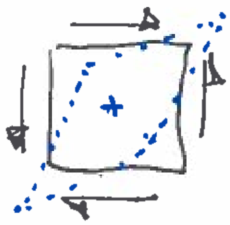
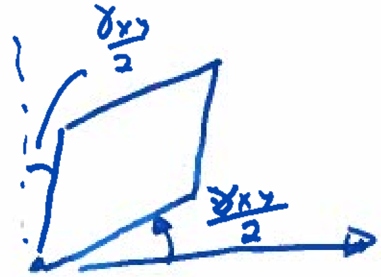
Shear Strain





shear strain

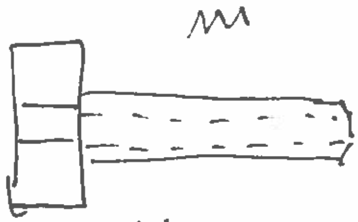
$$\frac{\gamma_{xy}}{2}$$

$$G\,\gamma_{xy} = \tau_{xy}$$

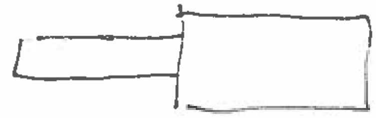
$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{bmatrix}$$

$$\sigma = \epsilon E$$

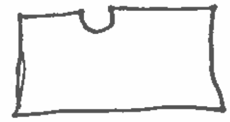
Stress Concentrations



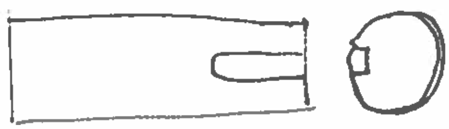
threads



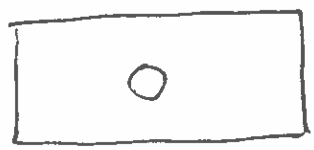
shoulders



notches



keyways



holes

Stress raisers: geometrical discontinuity that lead to non-standard stress

Stress concentrations: region in which stress raisers occur

Experimentally determine a stress concentration factor for a variety of shapes.

$$K_t = \frac{\sigma_{max}}{\sigma_o} \qquad K_{ts} = \frac{\tau_{max}}{\tau_o}$$

actual max stress and the concentration

nominal stress from geometry without the raiser

σ_o, τ_o: calculated by elementary stress equations

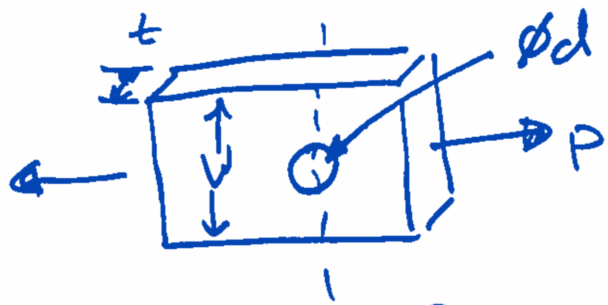
Some values K_t and $K_{\tau s}$ are found in Tables A-15 and A-16.

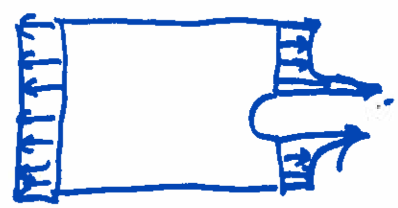
* <u>For static loading</u>*

Ductile materials: factors are generally not applied because the stress raisers have strengthening affect

Brittle materials: apply factor before checking strength

Hole in Plate



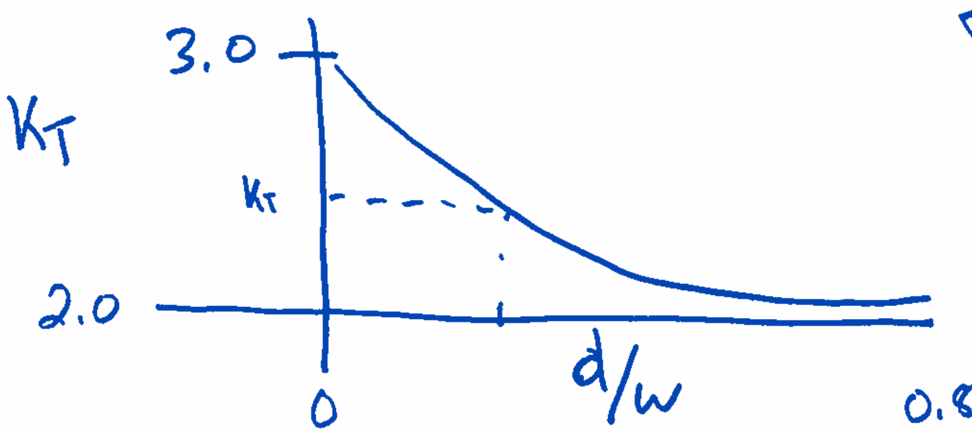


$$\sigma_0 = \frac{P}{A_{min}} = \frac{P}{(w-d)t}$$

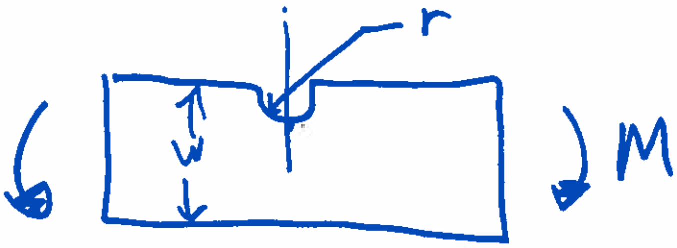
Table A-15!

$\sigma_{max} = K_T \sigma_0$

$2 \le K_T \le 3$

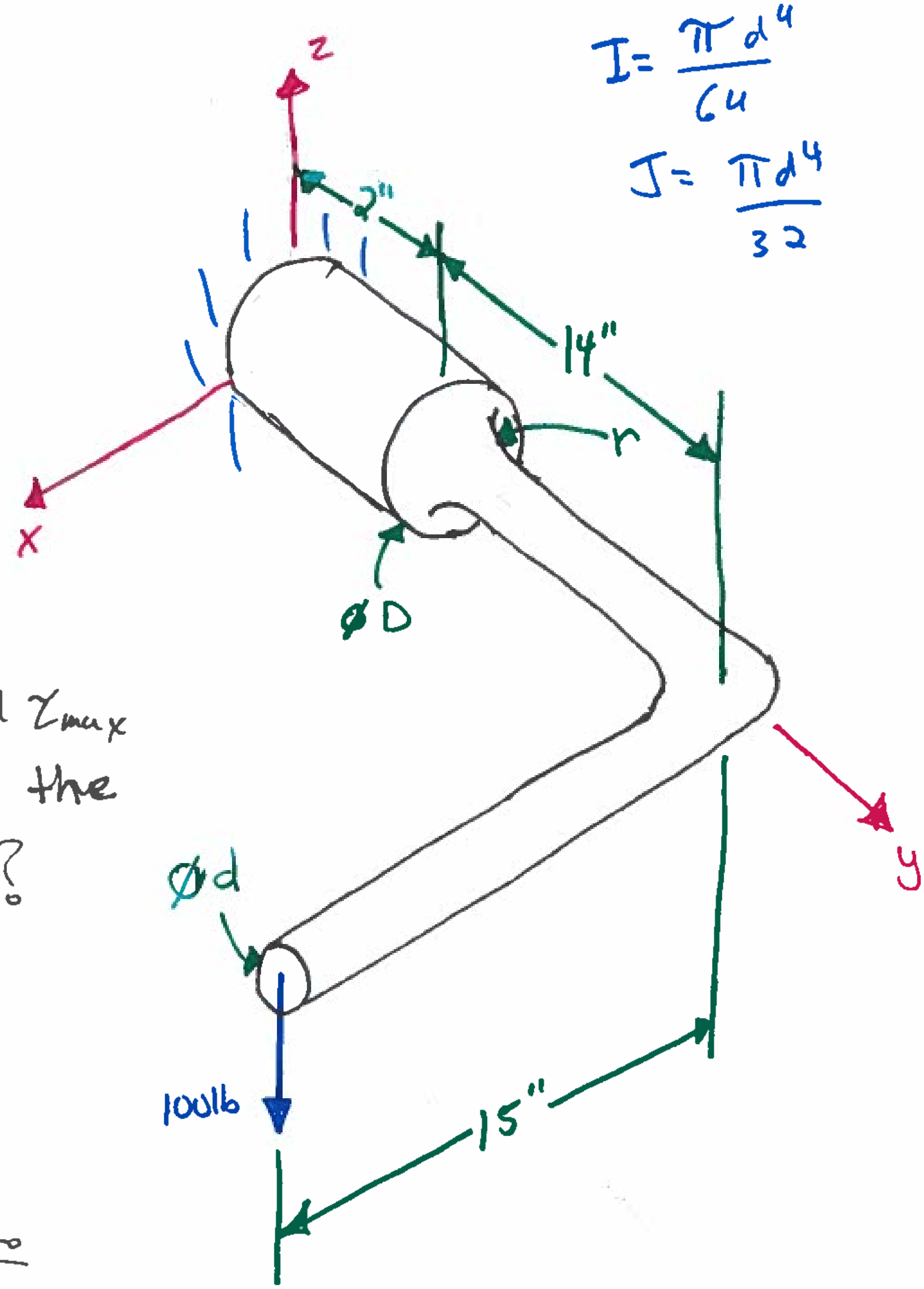


Notch



$$\sigma_0 = \frac{M\left(\frac{w-r}{2}\right)}{\frac{1}{12}t(w-r)^3}$$

$D = 2''$
$d = 1''$
$r = 0.125''$



$I = \frac{\pi d^4}{64}$

$J = \frac{\pi d^4}{32}$

What is σ_{max} and τ_{max} at the face with the fillet @ y = 2" ?

16" beam is long enough to ignore transverse shear.

Normal Stress due to bending

$$\sigma_{y0} = \frac{M\ c}{I} = \frac{64(1400\,in\cdot lb)(0.5'')}{\pi} = 14.26\ kpsi$$

$$M = (14'')(100\,lb) = 1400\,in\cdot lb$$

$$I = \frac{\pi d^4}{64} = \frac{\pi (1'')^4}{64} = \frac{\pi}{64}$$

$$c = \frac{1''}{2} = 0.5''$$

$$\tau_{yx0} = \frac{Tc}{J}$$

$$\tau_{yx0} = \frac{(15'')(100\,lb)(0.5'')}{\frac{\pi (1'')^4}{32}} = 7.6\ kpsi$$

For K_T and K_{TS} look to charts A-15-8 and A-15-9.

$$\frac{r}{d} = \frac{0.125''}{1''} = 0.125 \qquad K_T = 1.65$$

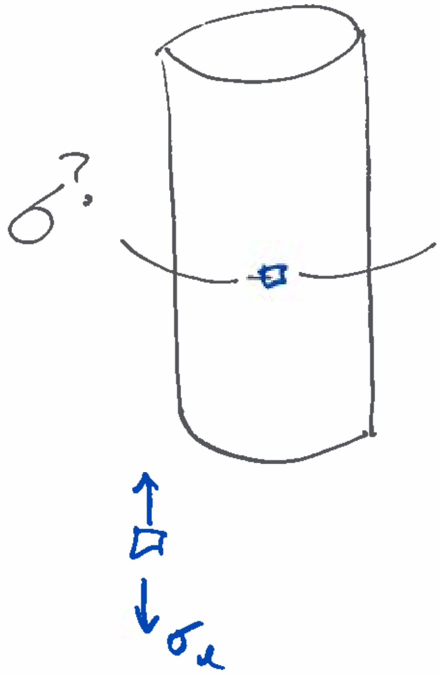
$$\frac{D}{d} = \frac{2''}{1''} = 2 \qquad K_{TS} = 1.4$$

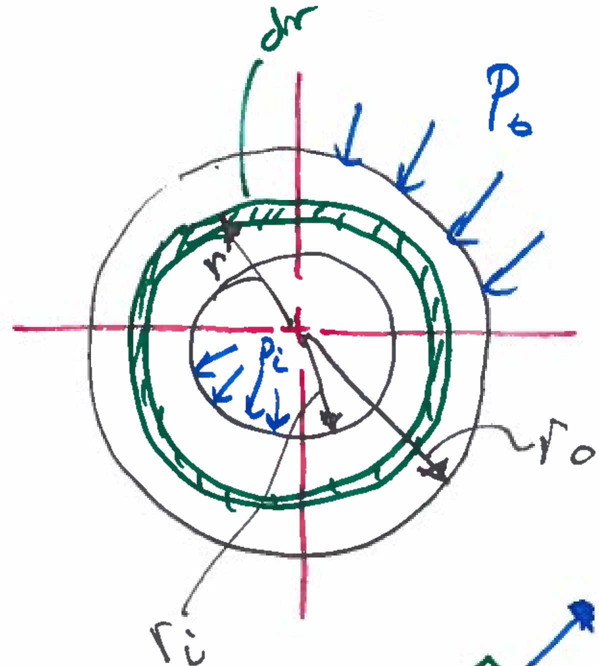
1.75 or 1.67 FIX!!

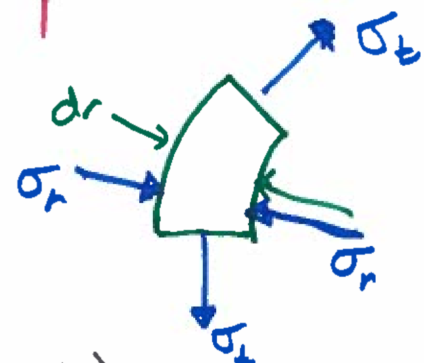
$$\sigma_{y(max)} = K_T \, \sigma_{yo} = (1.65)(14.26 \text{ Kpsi}) = 24 \text{ Kpsi}$$

$$\tau_{yx(max)} = K_{TS} \, \tau_{yxo} = (1.4)(7.6 \text{ Kpsi}) = 11 \text{ Kpsi}$$

Stress in pressure vessels







wall thickness $> \dfrac{\text{radius}}{10}$

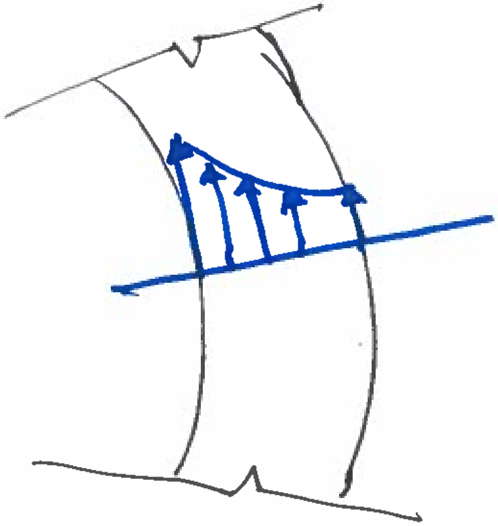
$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

r_o = outer radius

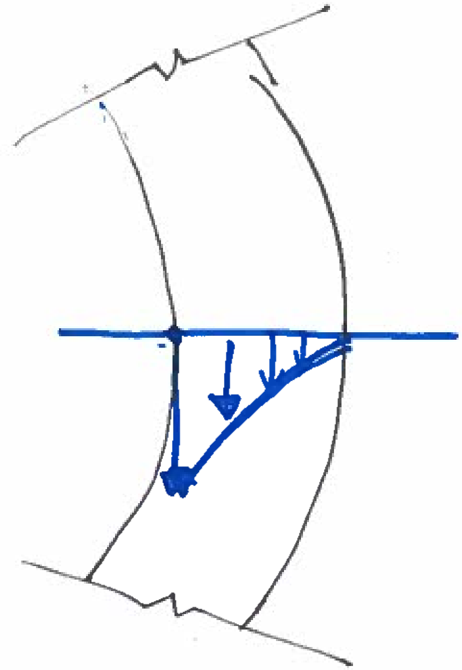
r_i = inner radius

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$ only valid away from the ends!



σ_t



σ_r

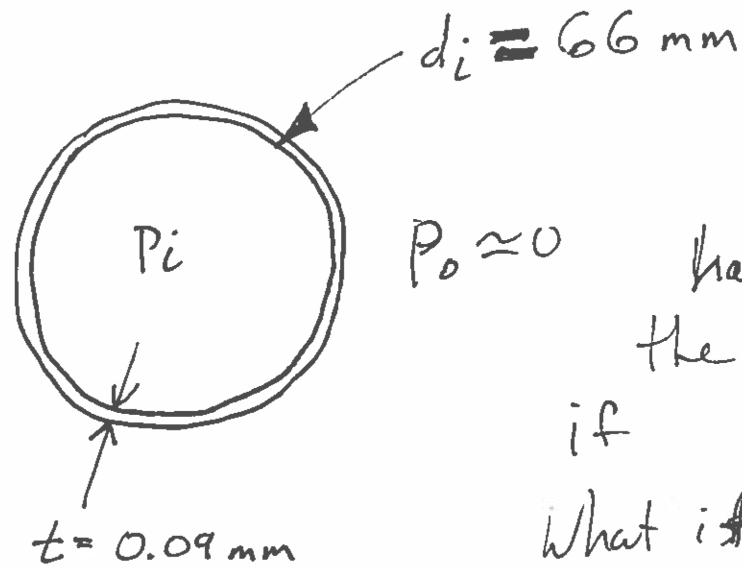
$\sigma_{inner} > \sigma_{outer}$

Special case $\Rightarrow$ $P_o = 0$

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

Soda Can Failure



What pressure does the inside have to reach for the can to fail if $S_{UT} = 215$ MPa?

What if $t = 0.03$ mm?

For thin walled vessels (hoop stress)

$$\sigma_{t_{max}} \approx \frac{P_i(d_i + t)}{2t} \quad \text{and} \quad \sigma_l = \frac{P_i d_i}{4t}$$

$$\sigma_r = 0$$

Note: an unshaken can @ room temp has a pressure of about 250KPa.

$t = 0.09\,mm$

$$\frac{2t\,S_{uT}}{(d_i+t)} = P_i = \frac{2(9E5\,m)(215E6\,Pa)}{(0.066m + 0.00009)} = \underline{585\,kPa}$$

tangential

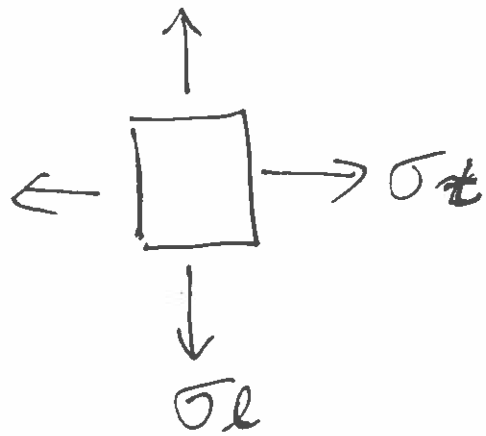
$$\frac{S_{ut}\,4t}{d_i} = \frac{(215E6Pa)(4)(9e-5\,m)}{0.066\,m} = \underline{1.2\,MPa}$$

longitudinal

$t = 0.03\,mm$

$P_i = 195\,kPa$ tangential

$P_i = 390\,kPa$ longitudinal



$0.09\,mm = 9e-5\,m$

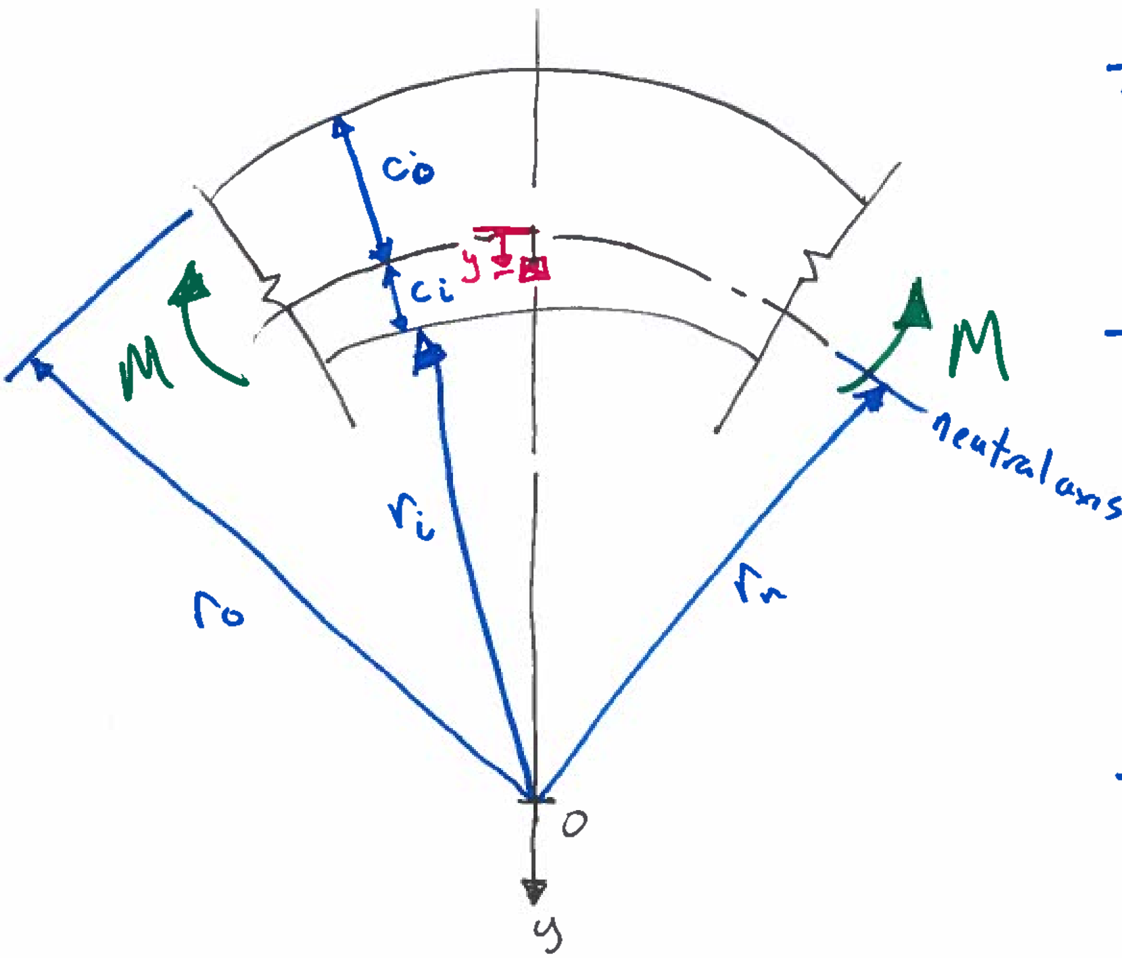
Curved Beams in Bending

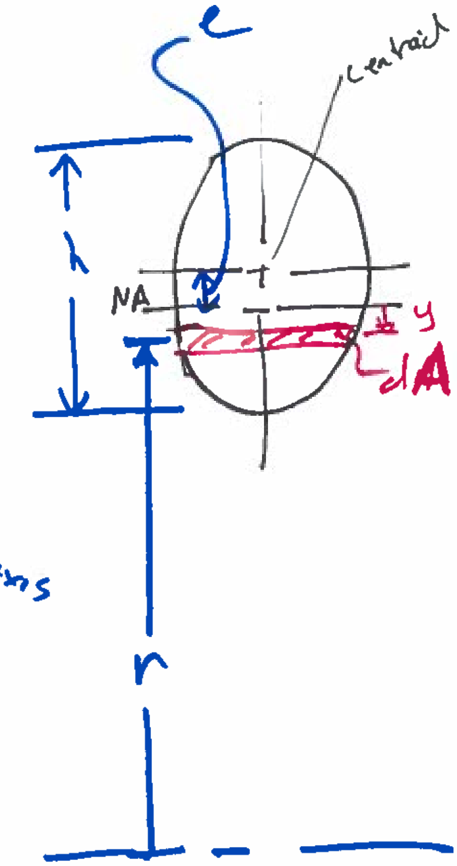
Can't use same analysis as straight beams!

Centroidal axis $\neq$ neutral axis

Assumptions

- cross section has axis of symmetry in the plane of bending
- Plane cross sections remain plane
- mod elasticity same in tension as compression


c_o
c_i
y
M
M
r_o
r_i
r_n
neutral axis
O
y


e
centroid
h
NA
y
dA
r

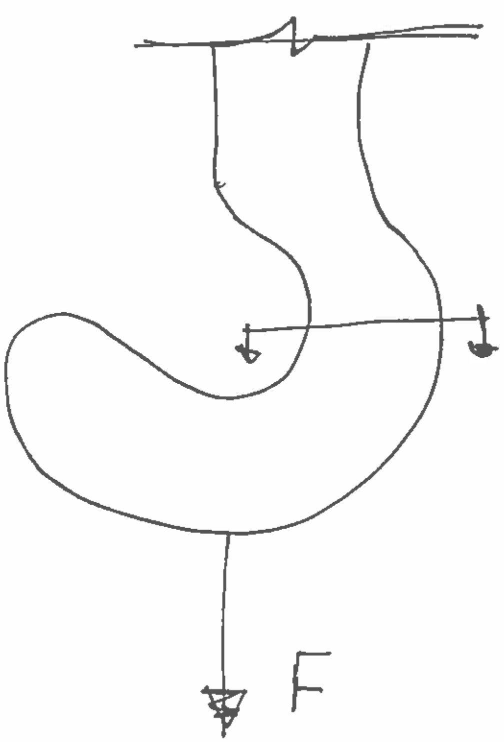
eccentricity

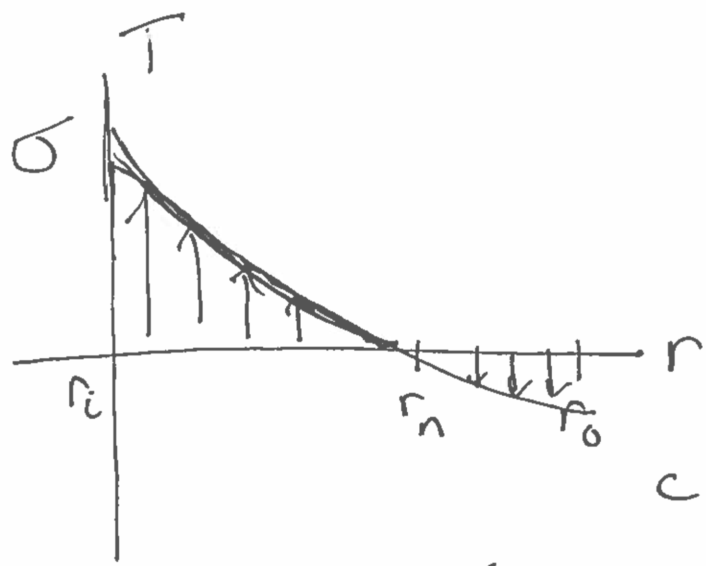
$$e = r_c - r_n \qquad e \approx \frac{I}{r_c A}$$

Normal Stress Distributions

$$\sigma(y) = \frac{My}{Ae(r_n - y)}$$ hyperbolic

$$\sigma_i = \frac{Mc_i}{Aer_i} \qquad \sigma_o = -\frac{Mc_o}{Aer_o}$$



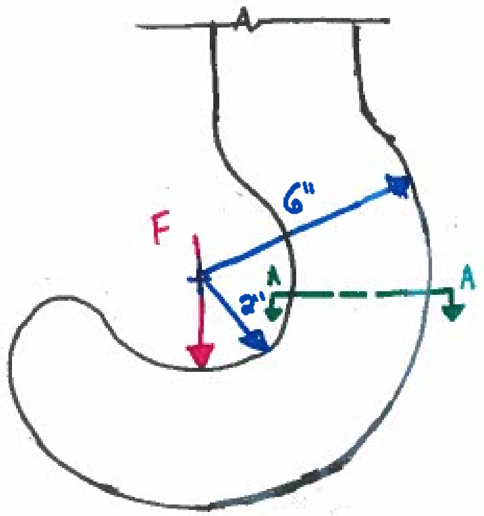


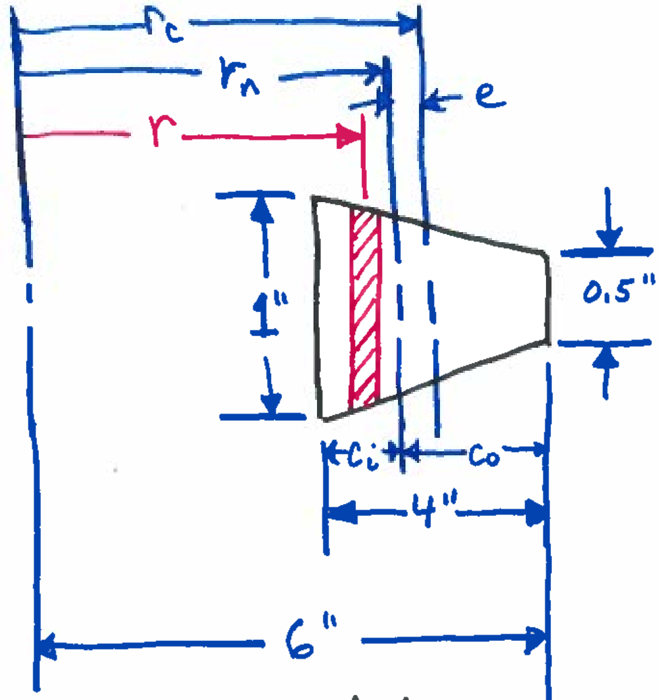
$$\sigma_i > \sigma_o$$

Curved Beam Example

This example mirrors Ex. 3-15 in the book but uses a more complex cross section. For the rectangular cross section $\sigma_i = 16.9$ ksi and $\sigma_o = -5.63$ ksi.

Fig 3-35



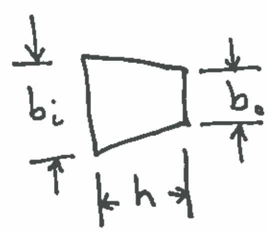


$F = 5000$ lb so $M = 20{,}000$ in.lb

Section A-A

From Table 3-4

$$r_c = r_i + \frac{h}{3}\,\frac{b_i + 2b_o}{b_i + b_o}$$



$r_i = 2"$

$h = 4"$

$b_i = 1"$

$b_o = 0.5"$

$r_o = 6"$

$$r_n = \frac{A}{b_o - b_i + [(b_i r_o - b_o r_i)/h]\ln(r_o/r_i)}$$

$$\sigma_i = \frac{M c_i}{A e r_i}$$

$$A = b_i h - \frac{(b_i - b_o)}{2} h$$

$$= (1")(4") - \frac{(1" - 0.5")}{2} 4" = 3 \text{ in}^2$$

$$r_n = \frac{3 \text{ in}^2}{0.5" - 1" [((1")(6") - (0.5")(2"))/4"]\ln\left(\frac{6"}{2"}\right)} = 3.44"$$

$c_i = r_n - r_i = 3.44" - 2" = 1.44"$ $\qquad$ $c_o = r_o - r_c = 2.22"$

$$r_c = 2" + \frac{4"}{3}\,\frac{1" + 2(0.5)"}{1" + 0.5"} = 3.78"$$

$e = r_c - r_n = 0.34"$

Max/min normal stress

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{5000\,\text{lb}}{3\,\text{in}^2} + \frac{(20{,}000\,\text{lb})(1.44")}{(3\,\text{in}^2)(0.34")(2")}$$

$\sigma_i = 15.8$ ksi

$$\sigma_o = -\frac{Mc_o}{Aer_o} + \frac{F}{A} = \frac{5000\,\text{lb}}{3\,\text{in}^2} - \frac{(20{,}000\,\text{lb})(2.22")}{(3\,\text{in}^2)(0.34")(6")}$$

$\sigma_o = -5.6$ ksi

Note that the max normal tensile stress was decreased slightly by choosing a cross section with more cross sectional area towards the inner surface.

$15.8\text{ ksi} < 16.9\text{ ksi}$