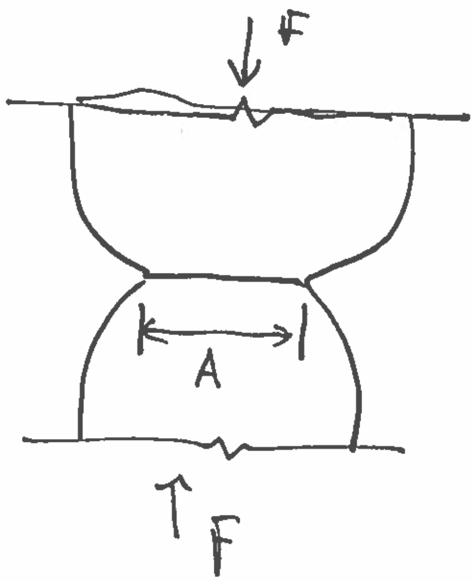
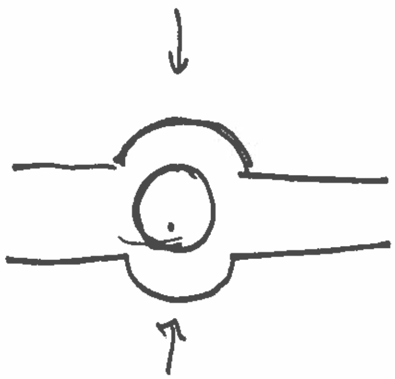
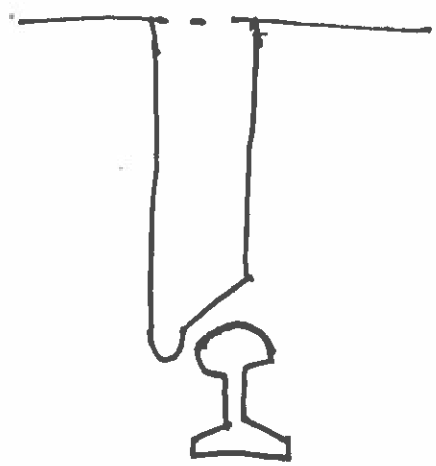
Contact Stresses



When two objects are pressed together a region of area contact develops.



roller bearing



train wheel

gear teeth

Theory developed Hertz 1882.

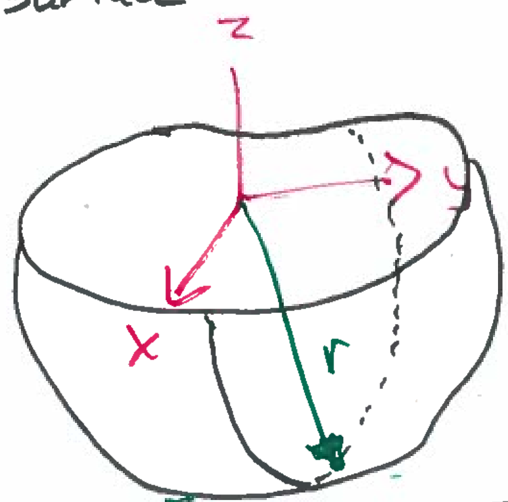
"Hertzian Contact Theory"

Assumptions

- loads are perpendicular to the surface
- no friction, smooth, continous surface
- small strain theory

General Case

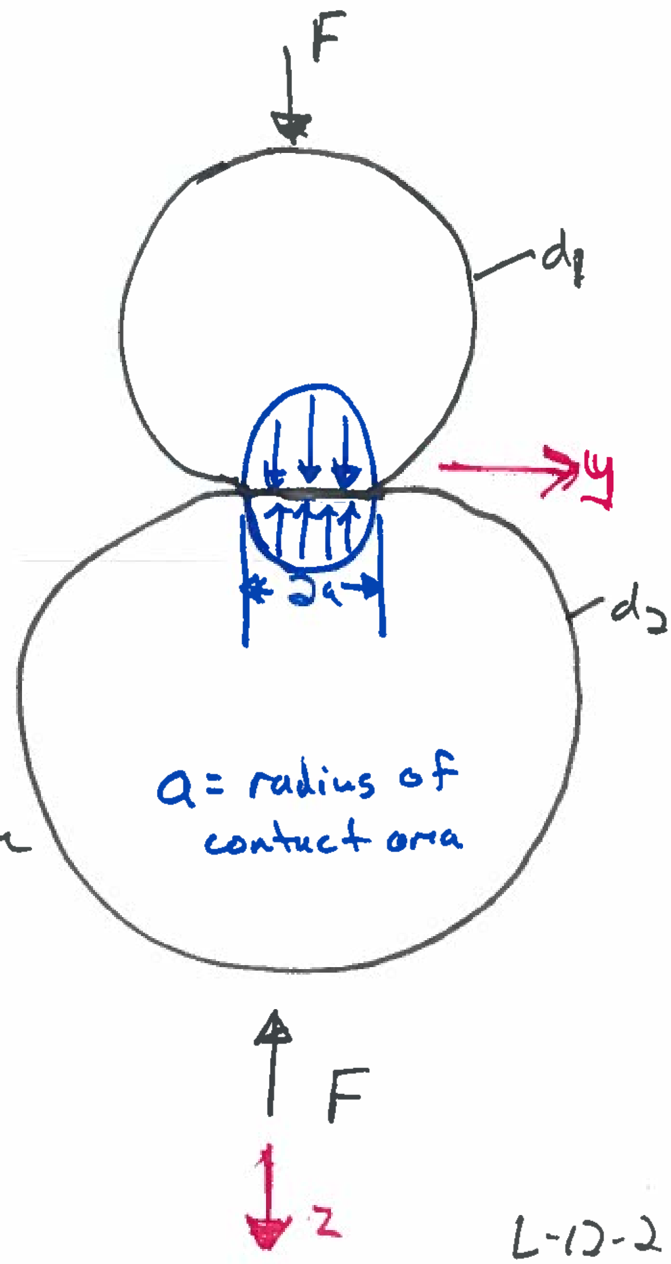
- general curved surface

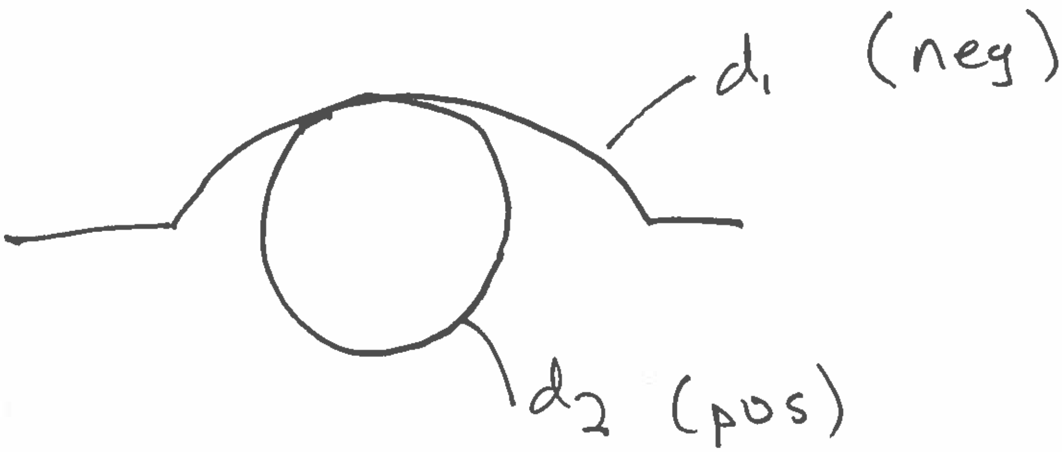


- circular area of contact
- hemispherical stress/pressure at the contact surface

Special Cases

Two Spheres

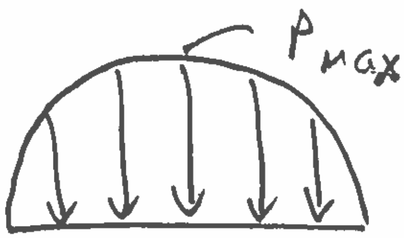




$$a = \sqrt[3]{\frac{3F}{8}\,\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}} \qquad \boxed{3\text{-}68}$$

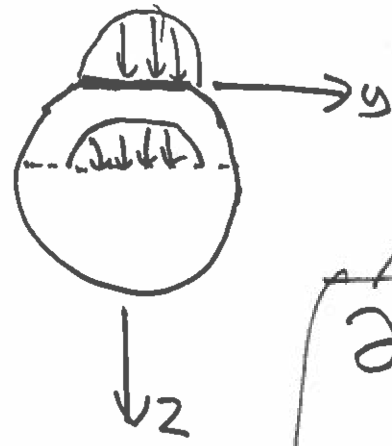
$d \Rightarrow \infty$ flat surface

$d < 0 \Rightarrow$ internal spherical shape 



$$p_{max} = \frac{3F}{2\pi a^2} \qquad \boxed{3\text{-}64}$$

The stress is a function of z.
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad ʒ$



partial two zee

$\partial \quad 2 \quad z$

$\emptyset \quad 0 \quad Ƶ$

Along z axis:

function of z

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -P_{max}\left[\left(1 - \left|\frac{z}{a}\right| \tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right]$$

$$\sigma_3 = \sigma_z = \frac{-P_{max}}{1+\frac{z^2}{a^2}}$$

Fig 3-37 Sphere contact stress as function of depth into sphere

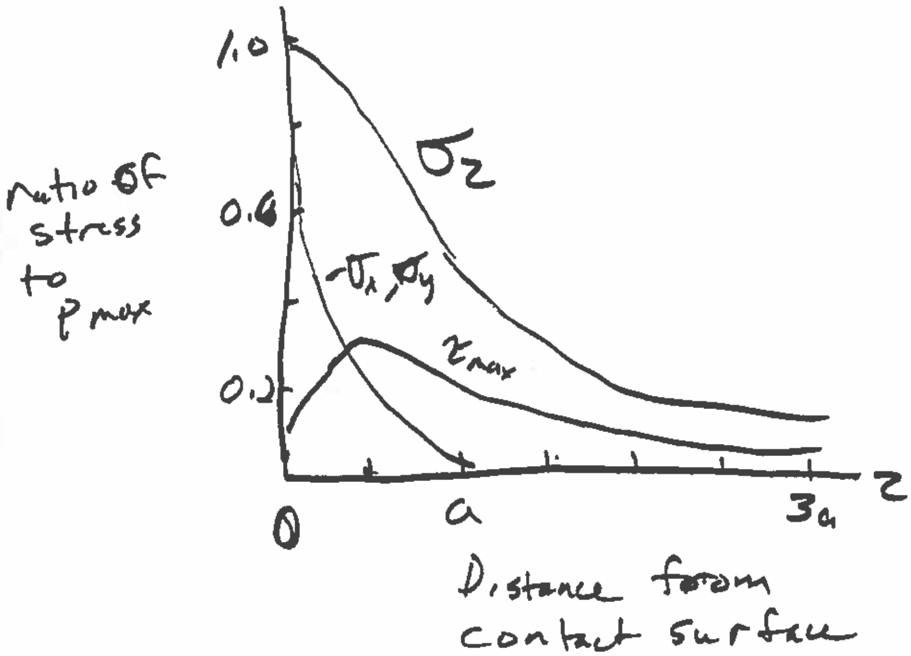
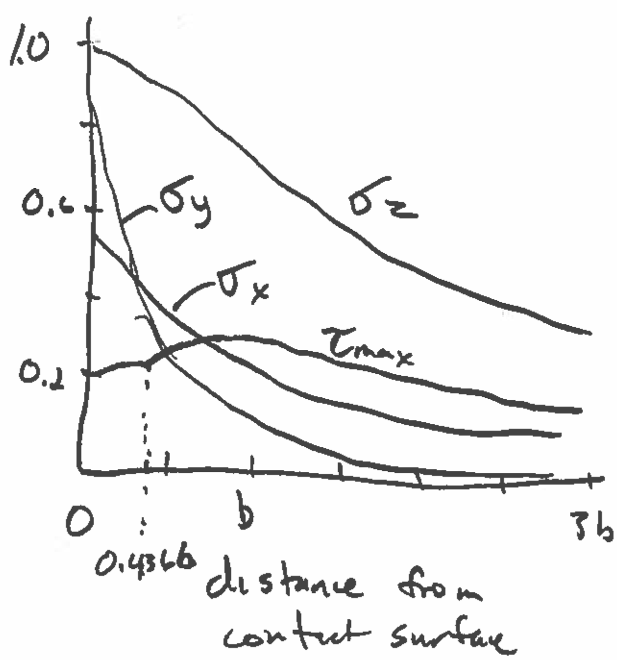


Fig 3-39 <u>Cylindrical</u>



spalling: flaking of material from the surface

brinelling: permanent indentation of a hard surface

pitting: spalling due to fatigue in roller bearings

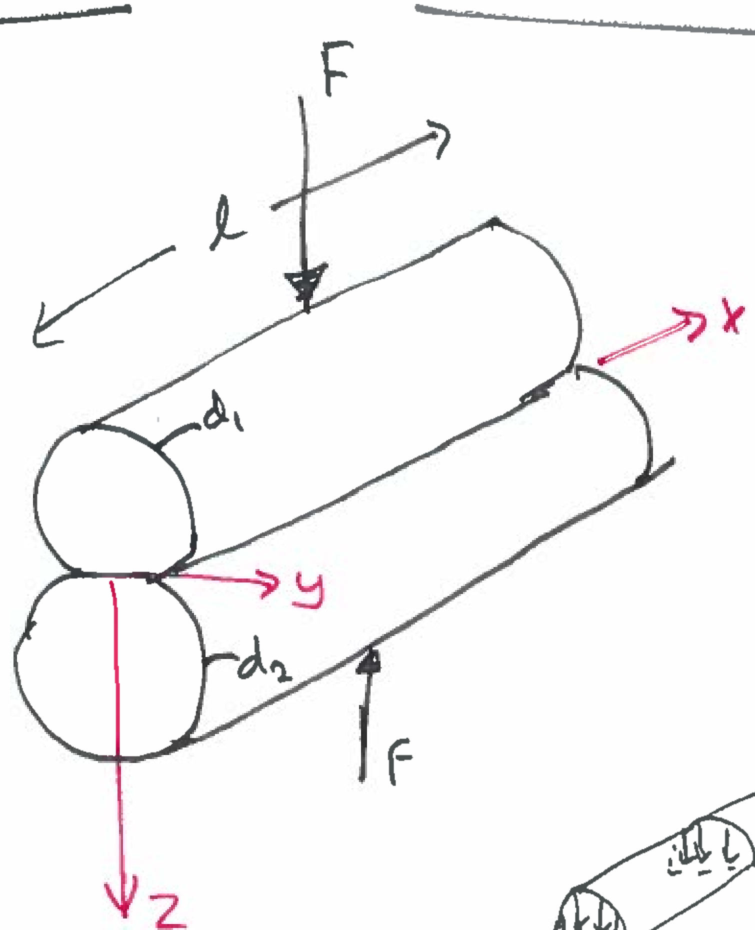
*see website resources page for examples

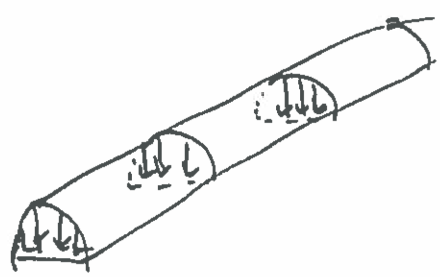
$$\sigma_1 = \sigma_2 \Rightarrow \tau_{1/2} = 0$$

max-shear below surface is attributed to spalling and pitting failures

$$\boxed{\tau_{max} = \tau_{1/3} = \tau_{2/3} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_2 - \sigma_3}{2}}$$

Special Case: Cylindrical Contact





$$b = \sqrt{\frac{2F}{\pi l}\,\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}} \qquad \boxed{3\text{-}73}$$

$$p_{max} = \frac{2F}{\pi b l}$$

$$\sigma_x = -2\nu p_{max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) \qquad \boxed{3\text{-}75}$$

$$\sigma_y = -p_{max}\left(\left[\frac{\sqrt{1+2\frac{z^2}{b^2}}}{\sqrt{1+\frac{z^2}{b^2}}}\right] - 2\left|\frac{z}{b}\right|\right) \qquad \boxed{3\text{-}76}$$

$$\sigma_3 = \sigma_z = \frac{-p_{max}}{\sqrt{1+\frac{z^2}{b^2}}} \qquad \boxed{3\text{-}77}$$