Castigliano's Theorem

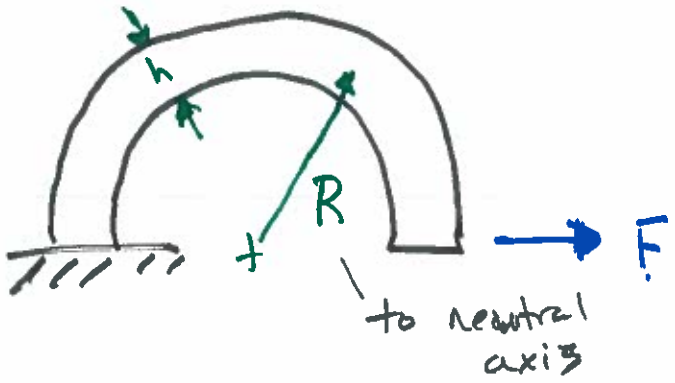
bending ..... strain energy

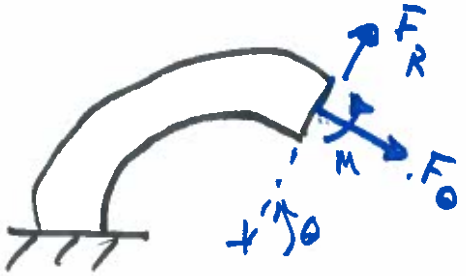
$$y_i = \frac{\partial U_i}{\partial Q_i} = \frac{\partial}{\partial Q_i}\left(\int \frac{M^2}{2EI}\,dx\right)$$

$$= \int \frac{\partial}{\partial Q_i}\left(\frac{M^2}{2EI}\right)dx$$

$$y_i = \int \frac{1}{EI}\left(M\frac{\partial M}{\partial Q_i}\right)dx$$

Curved Beams





4 components of strain energy

Moment

$$U_1 = \int \frac{M^2\,d\theta}{2A_e E}$$

Axial

$$U_2 = \int \frac{F_\theta^2 R\,d\theta}{2AE}$$

Tranverse Shear

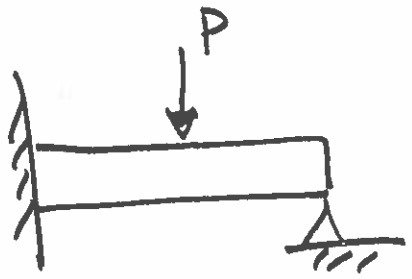
$$U_3 = \int \frac{C F_r^2 R\,d\theta}{2AG}$$

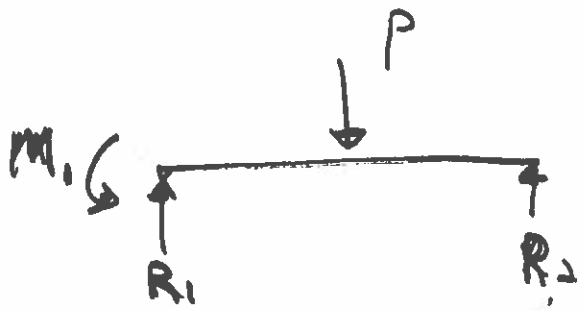
bending & Axial coupling term

$$U_4 = -\int \frac{M F_\theta \, d\theta}{AE}$$

$$U_{TOT} = U_1 + U_2 + U_3 + U_4$$

Static Indeterminate Problems

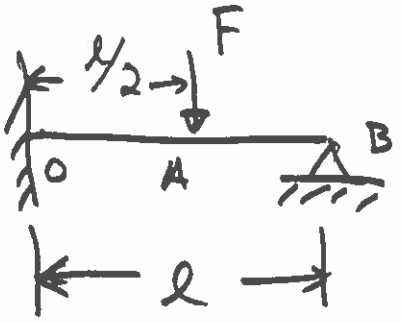


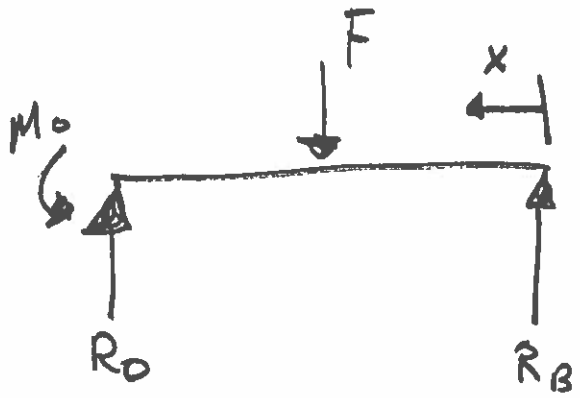


Procedure

1. Choose one the reactions as redundant.
2. Write the static equilibrium equations for the remaining reactions as functions of the applied loads and the redundant reactions.
3. Apply Castigliano's theorem to the total strain energy. $\frac{\partial U}{\partial R_{Unknown}} = 0$

Example 4-14





1. choose M_o to be redundant.

2. $R_O = \frac{F}{2} + \frac{M_o}{l}$ $\quad R_B = \frac{F}{2} - \frac{M_o}{l}$ $\quad \Sigma F=0$ $\quad \Sigma M=0$

3. $\Theta_o = 0 = \frac{\partial U}{\partial M_o}$

$$M = \left(\frac{F}{2} - \frac{M_o}{l}\right)x \qquad 0 \le x \le \frac{l}{2}$$

$$M = \left(\frac{F}{2} - \frac{M_o}{l}\right)x - F\left(x - \frac{l}{2}\right) \qquad \frac{l}{2} \le x \le l$$

$$\Theta_o = \int \frac{1}{EI}\left(M\frac{\partial M}{\partial M_o}\right)dx$$

$$\Theta_o = \frac{1}{EI}\left[\int_0^{l/2}\left(\frac{F}{2} - \frac{M_o}{l}\right)x\left(-\frac{x}{l}\right)dx + \int_{l/2}^{l}\left[\left(\frac{F}{2} - \frac{M_o}{l}\right)x - F\left(x - \frac{l}{2}\right)\right]\left(-\frac{x}{l}\right)dx\right] = 0$$

$$\left(\frac{F}{2} - \frac{M_0}{\ell}\right)\frac{\ell^3}{3} - \frac{F}{3}\left[\ell^3 - \left(\frac{\ell}{2}\right)^3\right] + \frac{F\ell}{4}\left[\ell^2 - \left(\frac{\ell}{2}\right)^2\right] = 0$$

Solve for M_0 (redundant reaction)

$$M_0 = \frac{3F\ell}{16}$$

$$R_0 = \frac{11F}{16} \qquad R_B = \frac{5F}{16}$$