Static Failure Theories

5-1 → 5-7

Fail: - distoration
- crack
- rupture
- yield
- etc

Best way to know when a mechanical element fail:

Test!!

- exact same geometry
- same material
- same conditions

Expensive !!

→ human safety
→ high volume manufacturing

} these are good reasons to test

Given tensile, compressive, and/or shear strength for a given material, how do we choose what to check given the state of stress? $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{zx}, \tau_{yz}$

$\sigma_1, \sigma_2, \sigma_3$ or $\tau_{1/2}, \tau_{2/3}, \tau_{1/3}$

Theories are broken ? two categories:

ε_f: strain at ~~ultimate~~ failure

Ductile

$\varepsilon_f \geq 0.05$

$S_{yt} = S_{yc} = S_y$

failure if stress > S_y

Brittle

$\varepsilon_f < 0.05$

S_{ut}, S_{uc}

Ductile Failure

- Maximum Shear Stress (MSS)
- Distortion Energy Theory (DE)
- Ductile Coulomb-Mohr Theory (DCM)

Maximum Shear Stress Theory

Yielding begins when τ_{max} at any point in a element equals or exceeds the τ_{max} in a tensile test specimen of the same material, surface finish, ambient temperature, ..., when it begins to yield.

~~If $(\tau_{max})_{general}$~~

If $(\tau_{max})_{general} \geq (\tau_{max})_{tensile\ specimen}$

$\Downarrow$

failure

For the tensile specimen $\tau_{max} = \dfrac{S_y}{2}$

3D State of stress:

$$\tau_{max} = \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

tensile yield strength

$$\therefore \tau_{1/3} \geq \frac{S_y}{2} \Rightarrow \sigma_1 - \sigma_3 \geq S_y$$

$$\sigma_1 - \sigma_3 = \frac{S_y}{n_{mss}} \Rightarrow \boxed{n_{mss} = \frac{S_y}{\sigma_1 - \sigma_3}}$$

2D plane stress

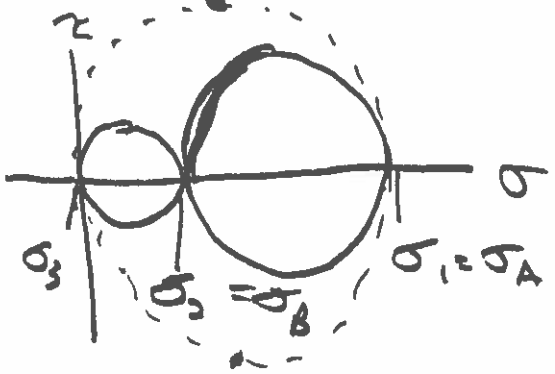
- one ~~principle~~ principal stress is zero

$\sigma_A, \sigma_B \Rightarrow \sigma_A \geq \sigma_B$

principal stresses

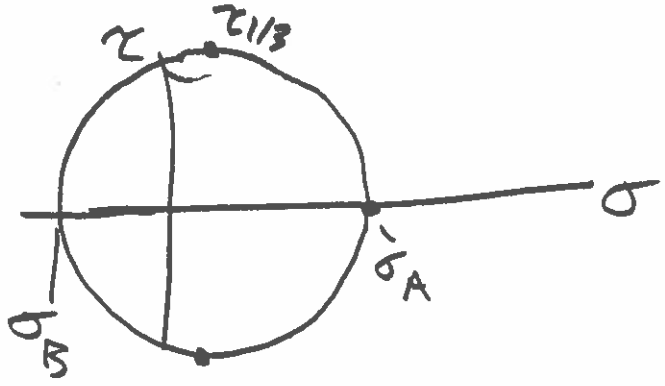
1) $\sigma_A \geq \sigma_B \geq 0$

$\boxed{\sigma_A \geq S_y \Rightarrow \text{failure}}$

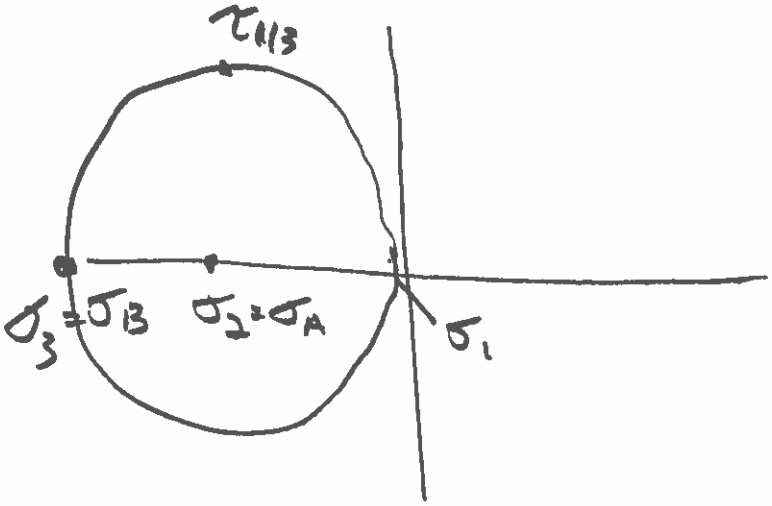


2) $\sigma_A \geq 0 \geq \sigma_B$

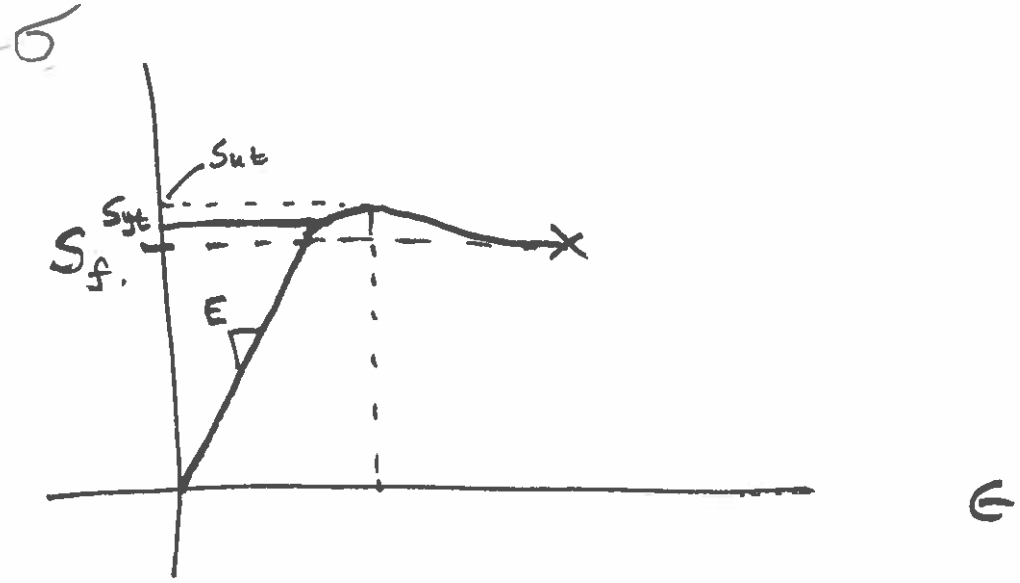
$\boxed{\sigma_A - \sigma_B \geq S_y}$

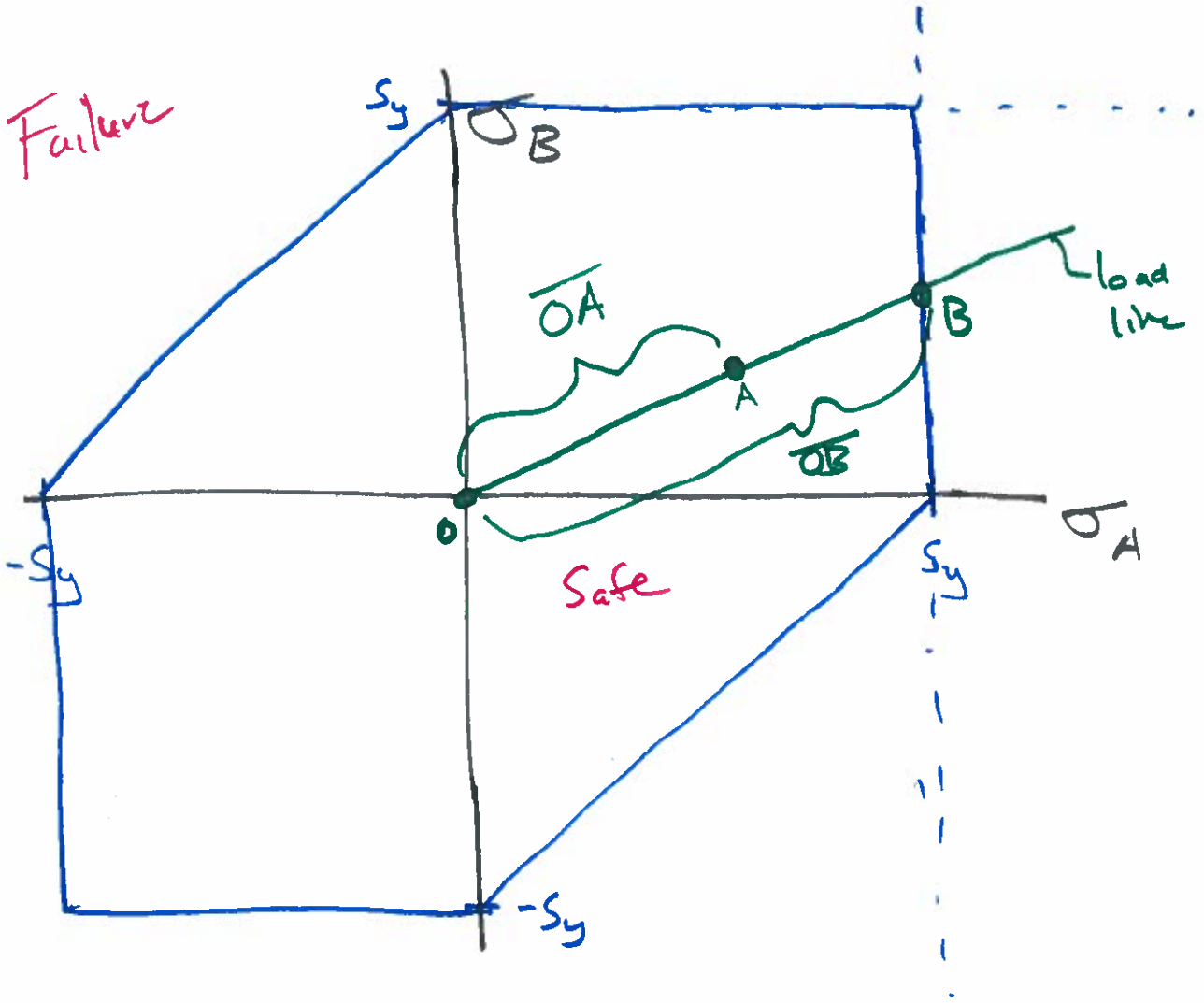


3) $0 \geq \sigma_A \geq \sigma_B$



$\sigma_B \leq -S_y$





$$n_{mss} = \frac{\overline{OB}}{\overline{OA}}$$

$$\sigma_A \geq \sigma_B$$

Distortion Energy Theory

$U = \frac{1}{2}\epsilon\sigma$ distortion strain energy per unit volume

$$\underbrace{\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]^{1/2}}_{\sigma' = \text{Von mises stress}} \geq S_y$$

constitutes failure

σ'

$$n_{de} = \frac{S_y}{\sigma'}$$

Plane Stress

$\sigma_1 = \sigma_A \quad \sigma_2 = \sigma_B$

$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}$$