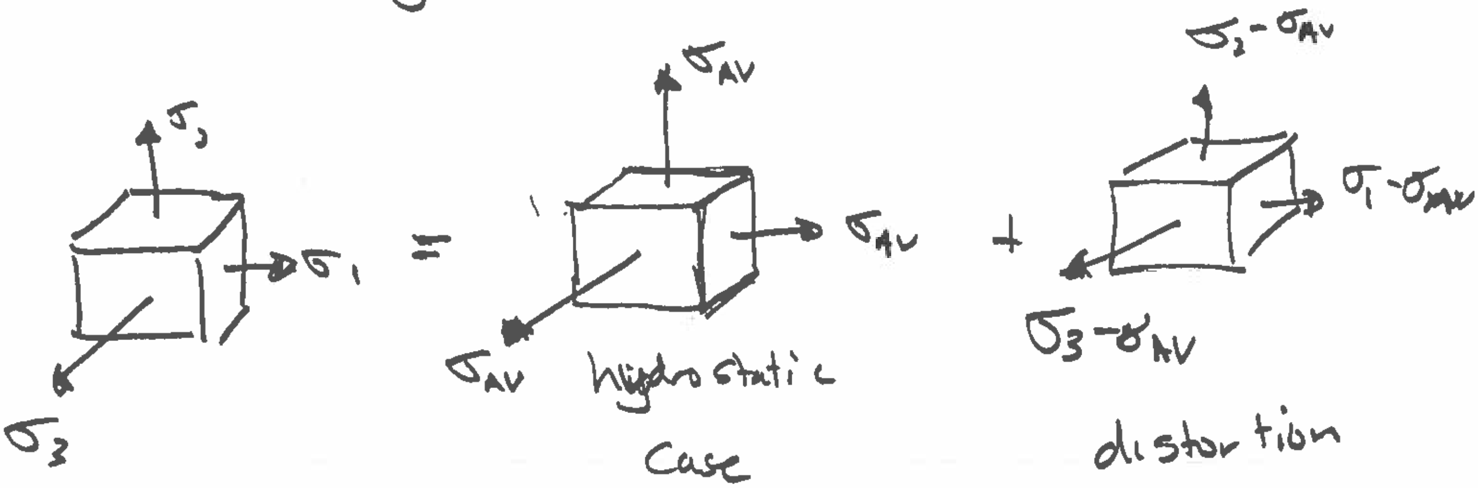
Distortion Energy Theory

Von mises stress

$$\sigma' \geq S_y$$

$$\sigma' = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}}$$

$$n_{de} = \frac{S_y}{\sigma'}$$



$$\sigma_{AVG} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

1D

$$u = \frac{1}{2}\epsilon\sigma$$

strain energy per unit volume

3D

$$u = \frac{1}{2}\left[\epsilon_1\sigma_1 + \epsilon_2\sigma_2 + \epsilon_3\sigma_3\right]$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

Volumetric strain energy

$$U_V = \frac{3}{2E}\,\sigma_{AV}^2\,(1-2\nu)$$ hydrostatic

distortion

$$U_d = U - U_V$$

$$= \frac{1+\nu}{3E}\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]$$

Failure will occure if distortion strain energy is greater than the distortion strain energy for tensile test specimen.

$$U_d \geqslant \frac{1+\nu}{3E}\,S_y^2$$

$$\sigma' \geqslant S_y$$

MSS $\quad S_{sy} = 0.5\, S_y$

shear ↙ $\quad$ ↙ tensile

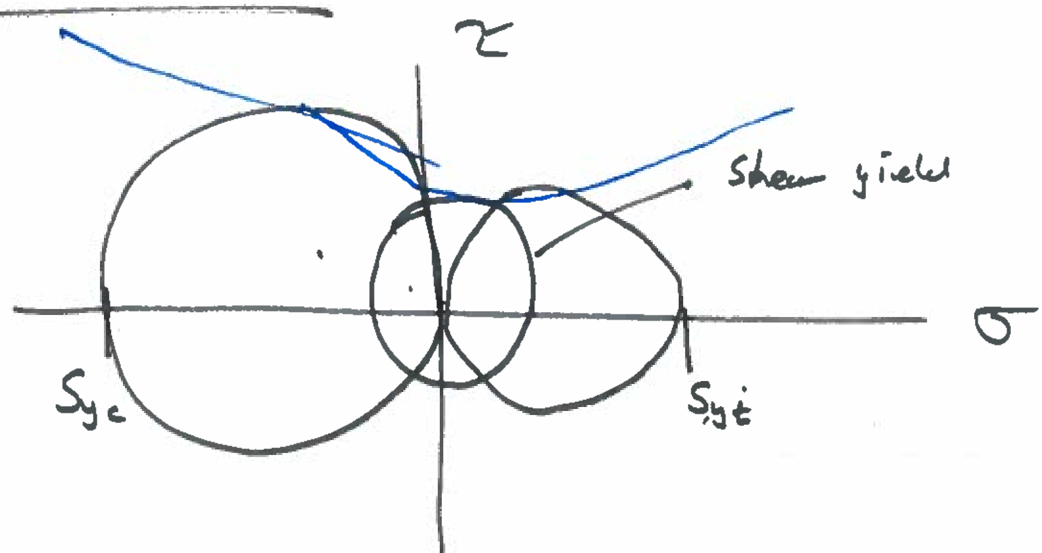
DE $\quad S_{sy} = 0.577\, S_y$

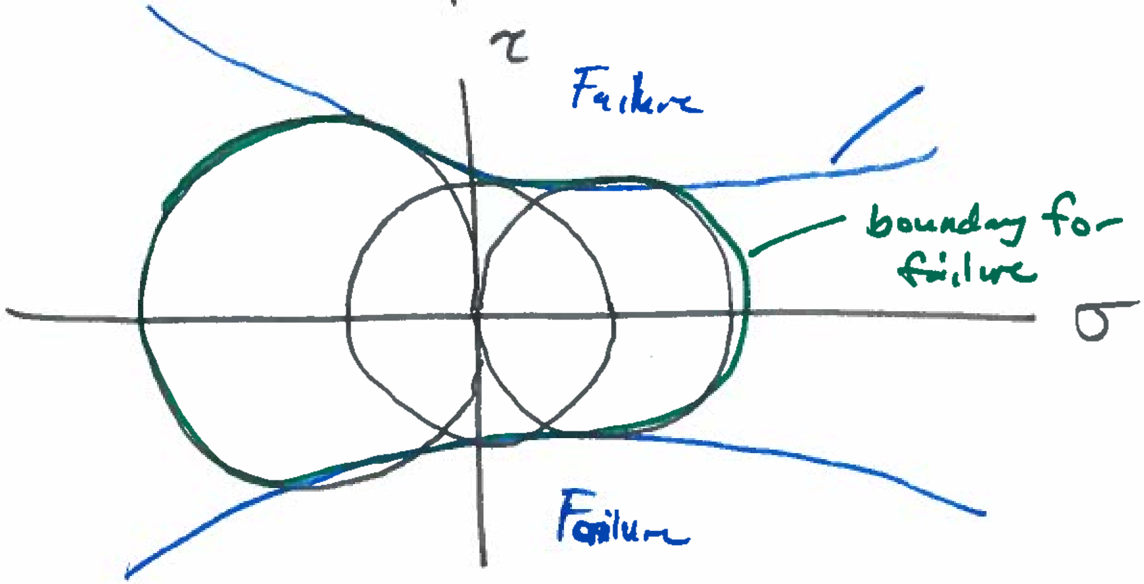
Coulomb-Mohr Theory

Some ductile materials:

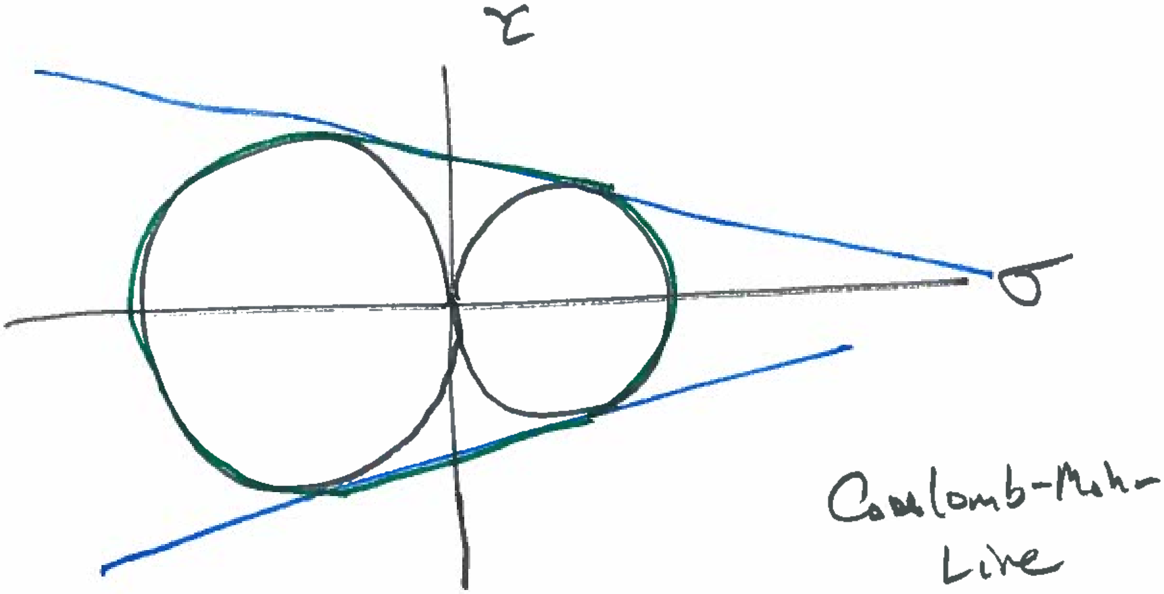
$$|S_{yt}| \neq |S_{yc}|$$

Mohr Theory



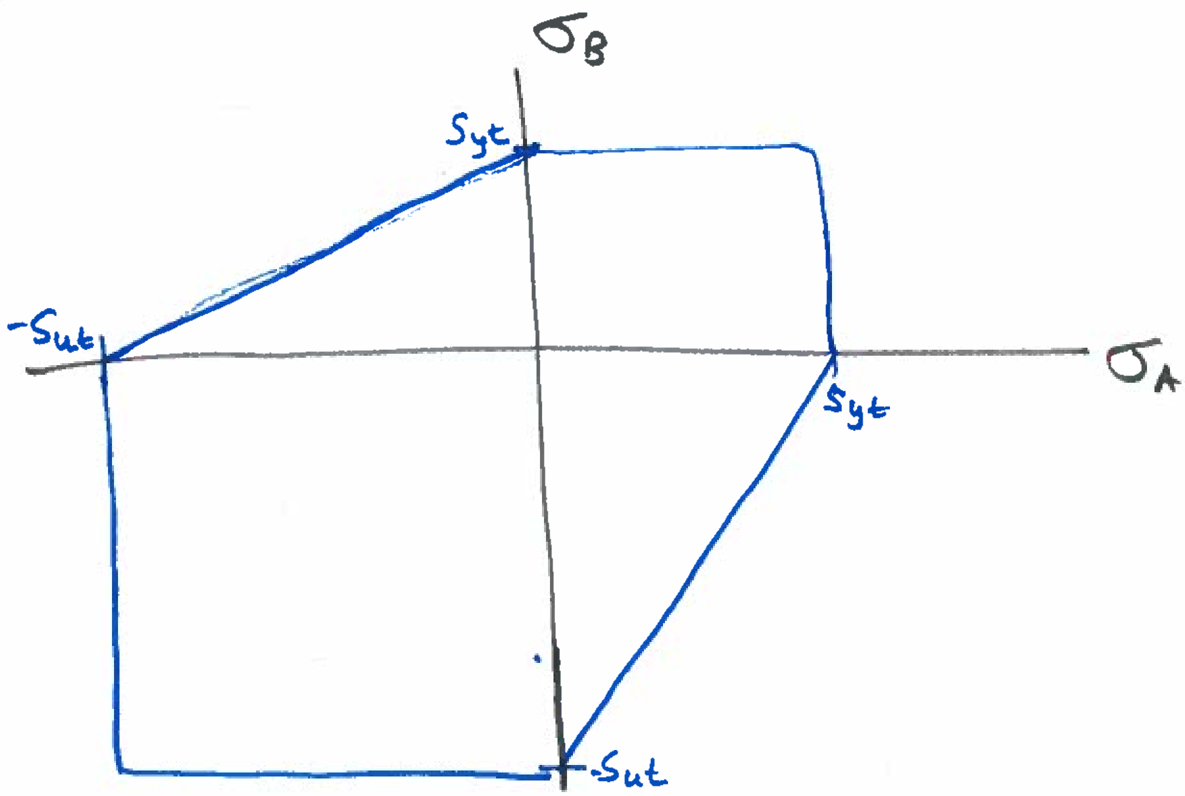


Coulomb - Mohr Theory



$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$

2D



Example

Steel $S_y = 295\,MPa$

Ductile

Given

$\sigma_x = -80\ MPa$, $\sigma_y = 30\,MPa$, $\tau_{xy} = 10\,MPa$

What are the factors of safety for MSS and DE?

Solution

$$\sigma_1 = \sigma_A = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - \tau_{xy}^2} = 30.9\ MPa$$

$$\sigma_3 = \sigma_B = \text{''} - \text{''} = -80.9\ MPa$$

$$\sigma_A \geq 0 \geq \sigma_B$$

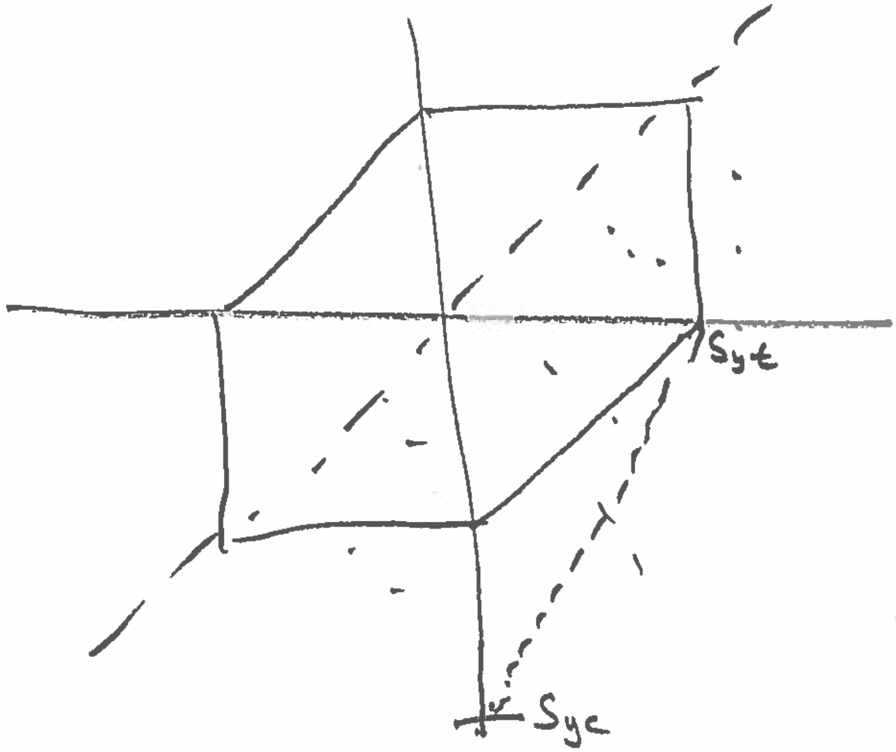
MSS

$$n_{MSS} = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{295\,MPa}{30.9 - (-80.9)} = 2.64$$

DE

$$\sigma' = \sqrt{\frac{\sigma_1^2 + (-\sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} = 99.7\ MPa$$

$$n_{de} = \frac{295}{99.7} = 2.95$$



But $S_{yt} = 295\,MPa$ and $S_{yc} = 320\,MPa$

Coulomb Mohr Theory

$$n_{dcm} = \left[\frac{\sigma_A}{S_{yt}} - \frac{\sigma_B}{S_{yc}}\right]^{-1} = \left[\frac{30.9\,MPa}{295\,MPa} - \frac{-80.9\,MPa}{320\,MPa}\right]^{-1}$$

$n_{dcm} = 2.79$

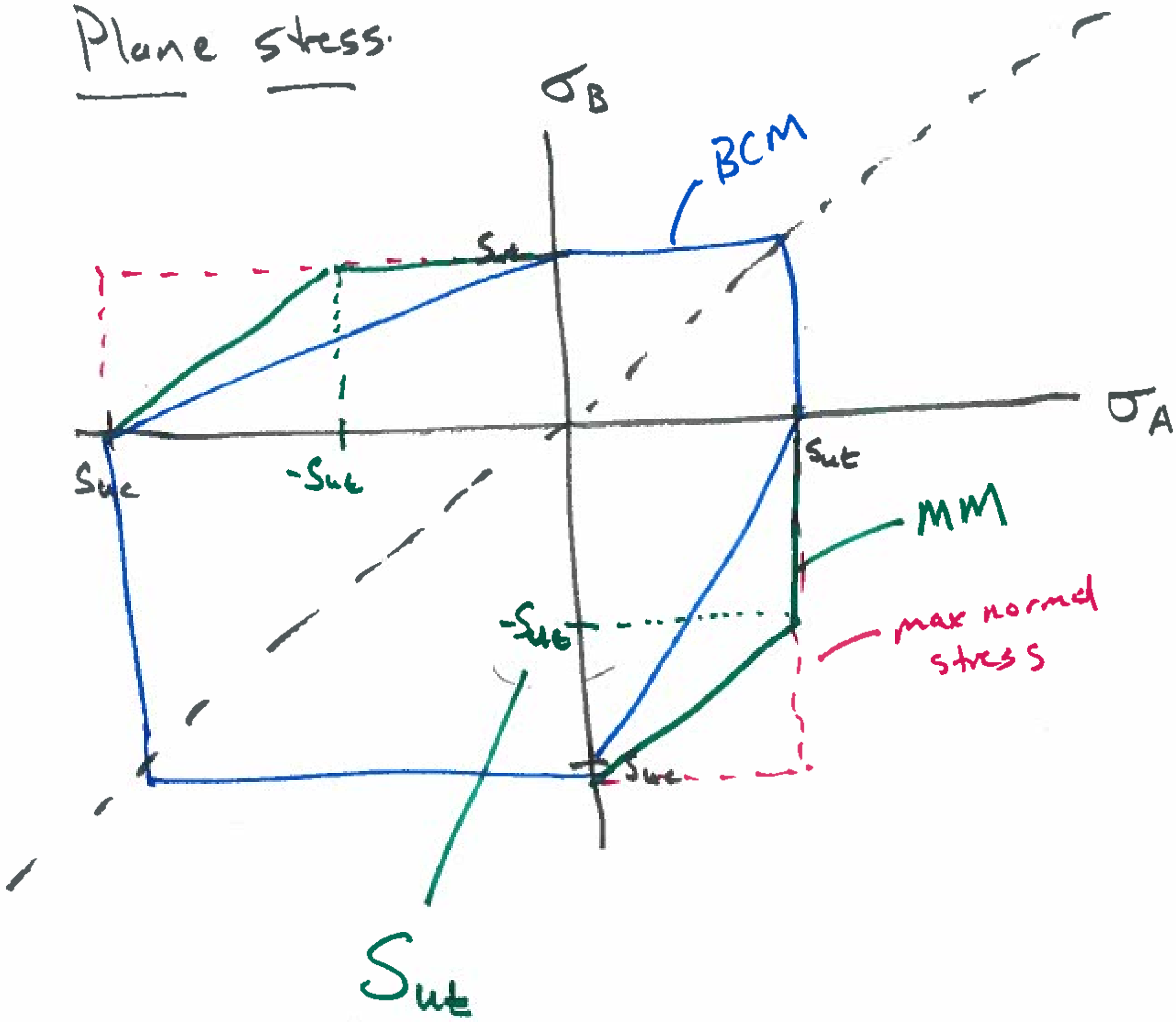
Failure in Brittle Materials

- Brittle fails due to fracture instead of yielding, in general
- Compression ultimate strength > tensile ultimate strength
- Empiral studies:
  - tension failure is ~~due~~ due to normals
  - compressive failure is due compressive normal stresses and shear stresses

Theories for static failure

- Max normal stress theory => ~~ignore~~ Not commonly used
- Brittle Coulomb-Mohr theory (BCM)
- Modified Mohr (MM)

Plane stress



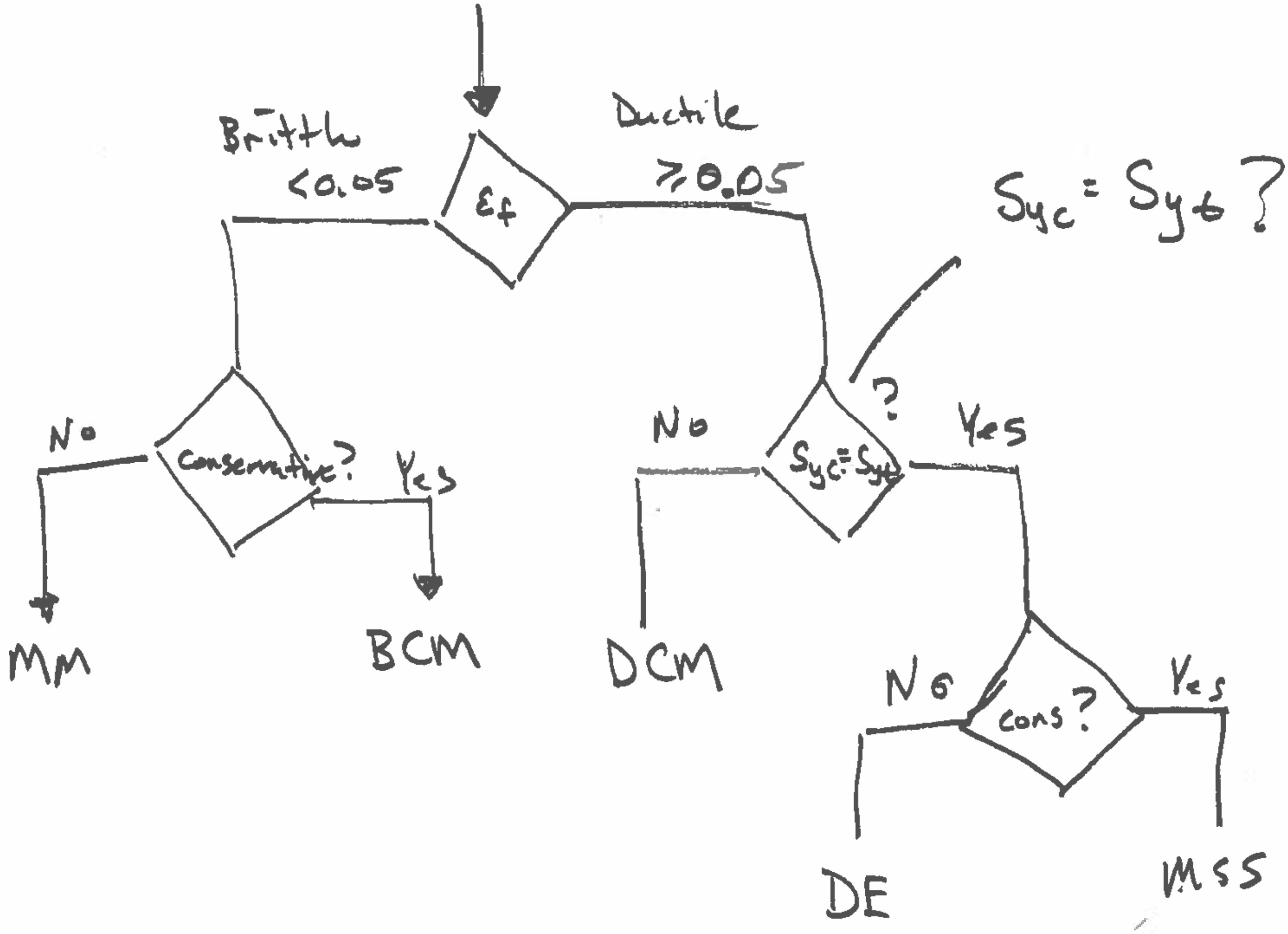
Modified Mohr

$$\sigma_A = \frac{S_{ut}}{n_{mm}} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$'' \qquad \sigma_A \geq 0 \geq \sigma_B \text{ and } \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$$

$$\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n_{mm}} \qquad \sigma_A \geq 0 \geq \sigma_B \text{ and } \left|\frac{\sigma_B}{\sigma_A}\right| > 1$$

$$\sigma_B = -\frac{S_{uc}}{n_{mm}} \qquad 0 \geq \sigma_A \geq \sigma_B$$


Brittle
<0.05
ε_f
Ductile
≥0.05
$S_{yc} = S_{yt}$?
No
Conservative?
Yes
MM
BCM
No
?
$S_{yc} = S_{yt}$
Yes
DCM
No
cons?
Yes
DE
MSS