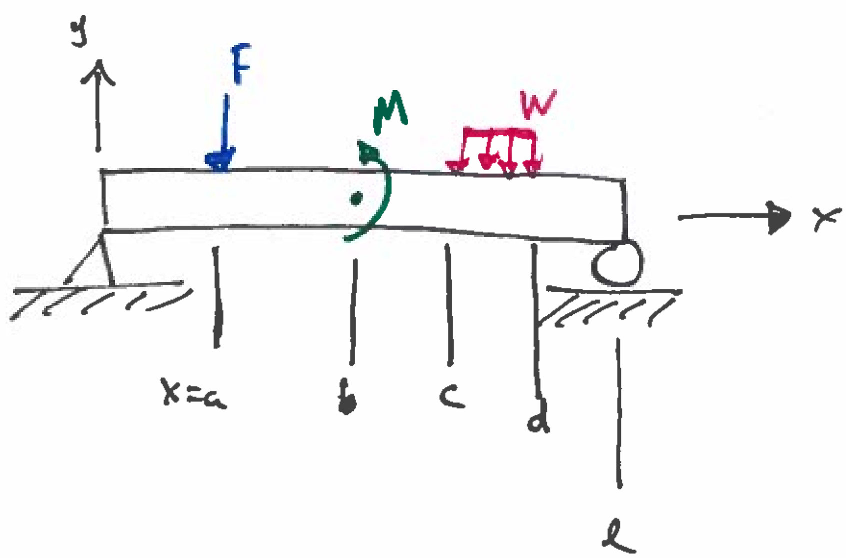
Example

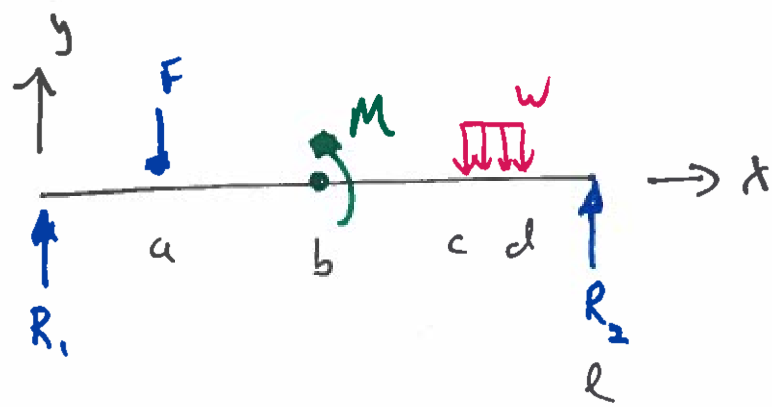


Known: $F, M, W, a, b, c\ d, \ell$

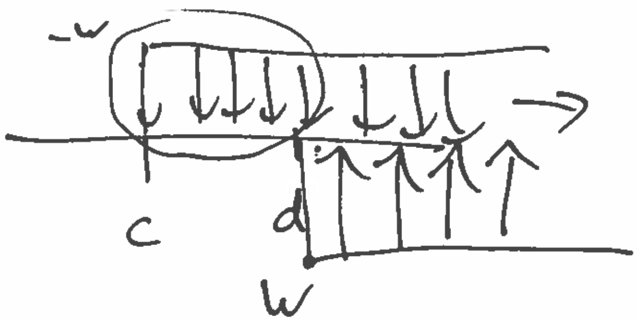
Find: - reactions @ $x=0, x=\ell$

- shear and bending moment diagrams.

FBD



$$q(x) = R_1\langle x-0\rangle^{-1} - F\langle x-a\rangle^{-1} - M\langle x-b\rangle^{-2}$$
$$-w\langle x-c\rangle^{0} + w\langle x-d\rangle^{0} + R_2\langle x-\ell\rangle^{-1}$$



$$V(x) = R_1\langle x-0\rangle^{0} - F\langle x-a\rangle^{0} - M\langle x-b\rangle^{-1}$$
$$-\frac{w}{1}\langle x-c\rangle^{1} + \frac{w}{1}\langle x-d\rangle^{1} + R_2\langle x-\ell\rangle^{0}$$

$$M(x) = R_1\langle x-0\rangle^{1} - F\langle x-a\rangle^{1} - M\langle x-b\rangle^{0}$$
$$-\frac{w}{2}\langle x-c\rangle^{2} + \frac{w}{2}\langle x-d\rangle^{2} + R_2\langle x-\ell\rangle^{1}$$

Reactions at $x=0$ and $x=\ell$?

$V(\ell^+)=0$

$M(\ell^+)=0$

$$0 = V(\ell^+) = R_1\langle \ell^+\rangle^0 - F\langle \ell^+ - a\rangle^0 - M\langle \ell^+ - b\rangle^{-1}$$
$$- w\langle \ell^+ - c\rangle^1 + w\langle \ell^+ - d\rangle^1 + R_2\langle \ell^+ - \ell\rangle^0$$
$$R_2\langle \ell^+ - \ell\rangle^0$$

$$0 = R_1 - F - w(\ell - c) + w(\ell - d) + R_2 \cdot 1$$

$$R_1 = F + w(\ell - c) - w(\ell - d) - R_2$$

$$0 = M(\ell^+) = R_1 \ell - F(\ell - a) - M - \frac{w}{2}(\ell - c)^2$$
$$+ \frac{w}{2}(\ell - d)^2 + \quad R(\ell - \ell)^1$$
$$+ 0$$

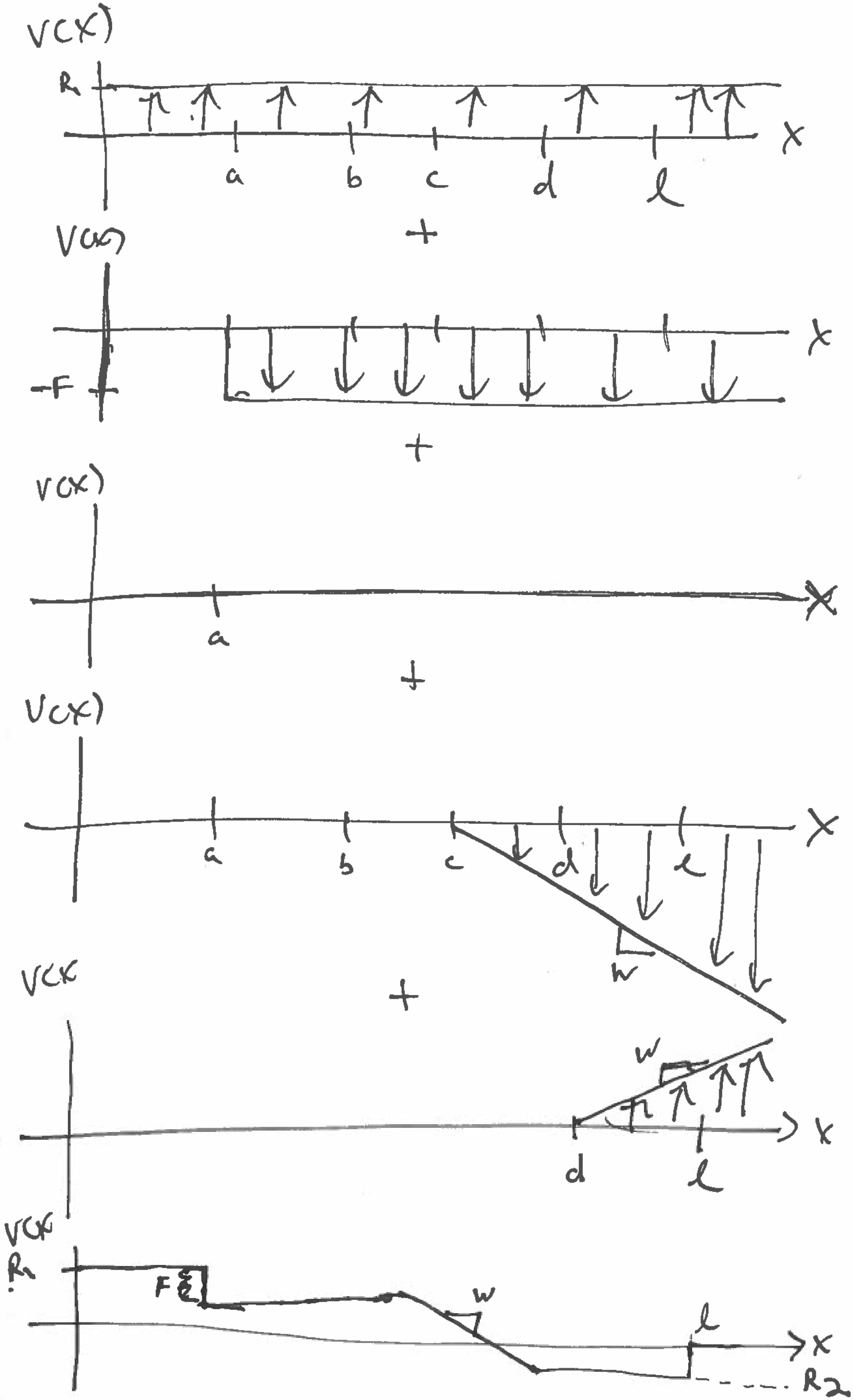
$$R_2 = \text{something}$$

Why R_2 stays in shear equation

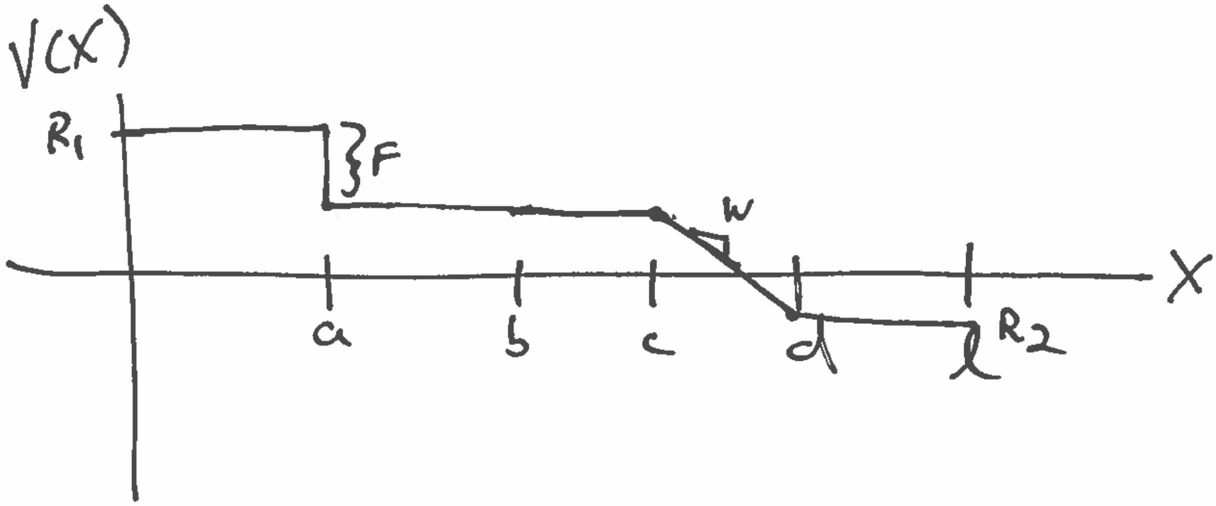
$$R_2\langle x - \ell\rangle^0$$

$$R_2\langle \ell^+ - \ell\rangle^0$$
$$x > a$$

$$R_2 \cdot 1$$



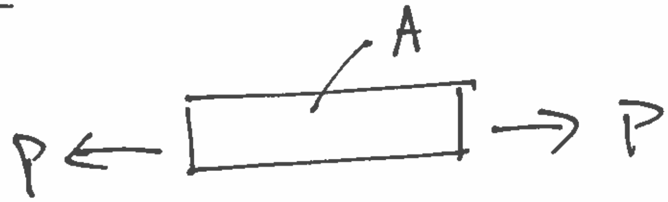
L4


V(X)
R1
F
W
X
a
b
c
d
l
R2

Normal Stress

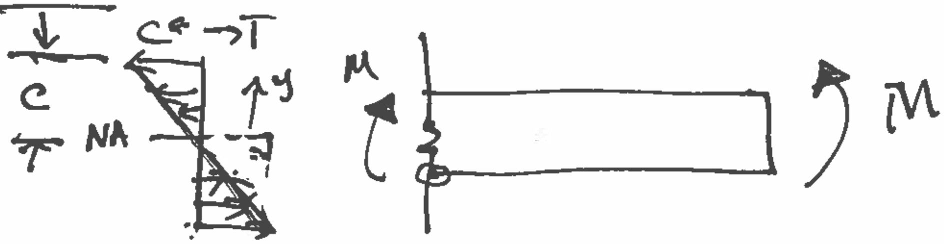
$$\sigma = \frac{P}{A}$$

A – cross section



Bending Stress

$$\sigma = \frac{My}{I}$$



I – Second moment of area

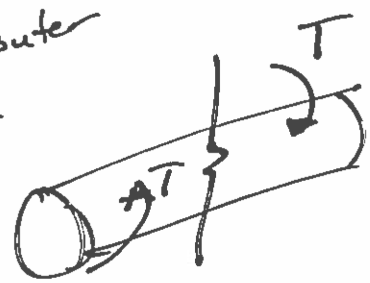
$$\sigma_{max} = \frac{Mc}{I}$$

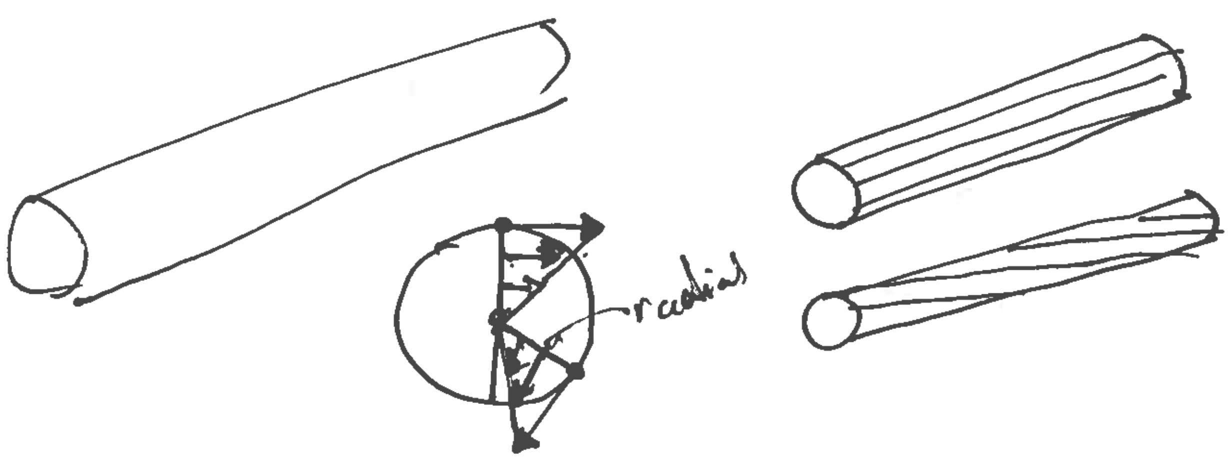
Torsional Stress

$$\tau_{max} = \frac{Tr}{J}$$

r – radius to outer surface

J – polar second moment of area



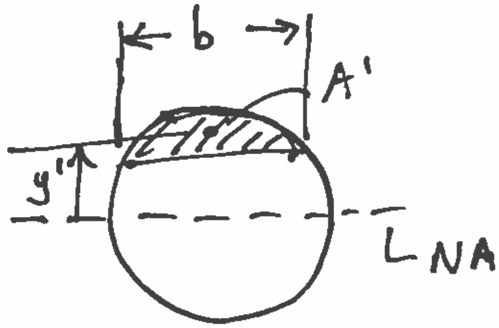


Transverse Shear Stress

$$\tau = \frac{VQ}{Ib}$$

V = Shear force

I = second moment of area of entire cross section



b = width at point of interest

Q : $\int y \, dA' = \bar{y}' A'$

A' = area above point of interest

$\bar{y}'$ = neutral axis to the centroid of A'

If $\frac{\text{beam length}}{\text{beam height}} > 10$

$\Downarrow$

$\tau \Rightarrow$ negligible