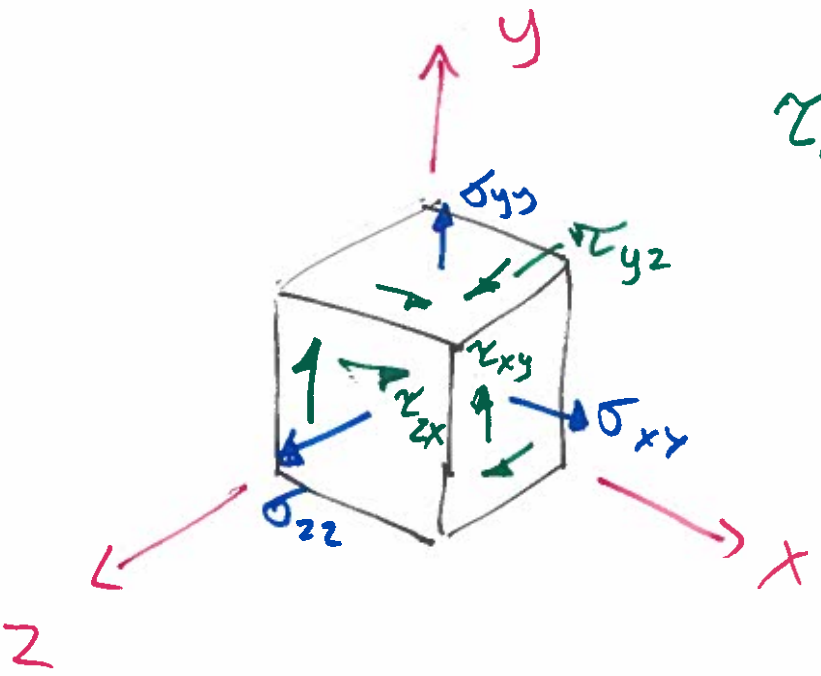
LECTURE 8

Friday, October 13, 2016



$$\tau_{ij} = \tau_{ji}$$

Cauchy Stress Tensor

2nd order tensor

6 unique elements

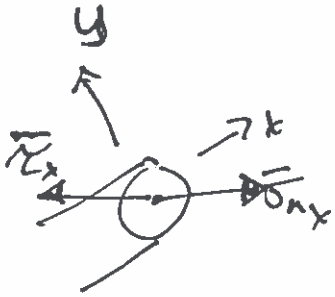
$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\bar{\sigma} = \begin{bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\sigma}_z \end{bmatrix} = \sigma \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix}$$

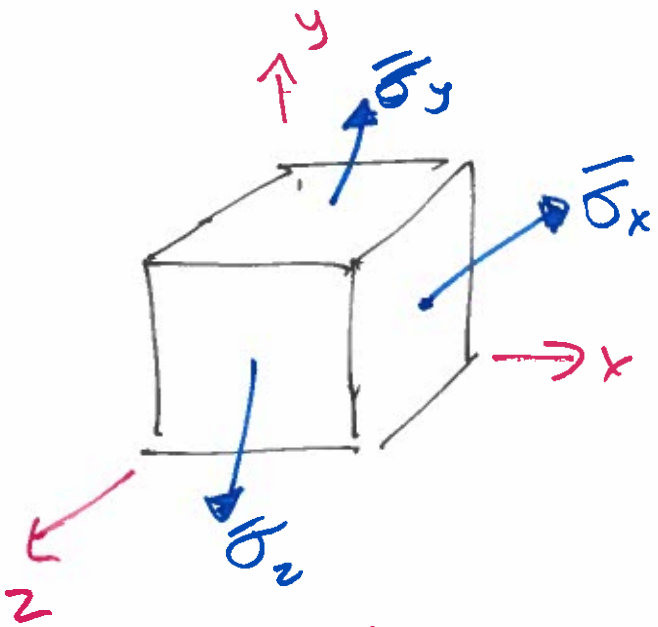
$$\bar{\sigma}_x = \underbrace{\sigma_{xx}\,\hat{i}}_{\bar{\sigma}_{nx}} + \underbrace{\tau_{xy}\,\hat{j} + \tau_{xz}\,\hat{k}}_{\bar{\tau}_x}$$

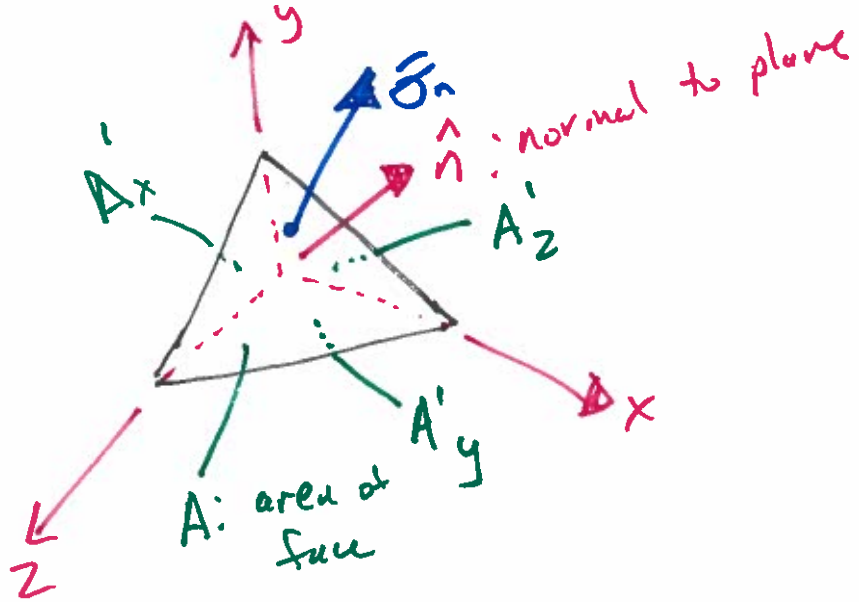
$\bar{\sigma}_{nx}$ normal

$\bar{\tau}_x$ shear



Computing stress on arbitrary cross sectional plane





$$\hat{n} = l\hat{i} + m\hat{j} + n\hat{k}$$

Tetrahedron has to be in equilibrium

$$\sum \bar{F} = 0 = A\bar{\sigma}_n - A'_x\bar{\sigma}_x - A'_y\bar{\sigma}_y - A'_z\bar{\sigma}_z$$

project A onto the coordinate planes

$A'_x = lA \quad A'_y = mA \quad A'_z = nA$

→ Contains normal & shear stress

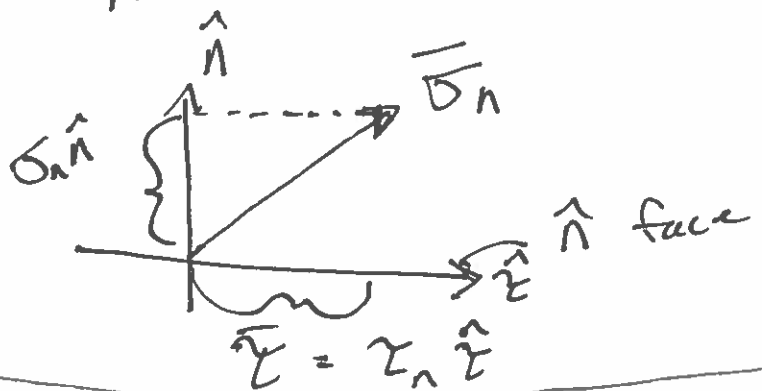
$$\therefore \quad \bar{\sigma}_n = l\bar{\sigma}_x + m\bar{\sigma}_y + n\bar{\sigma}_z$$

$$\bar{\sigma}_x = \sigma_{xx}\hat{i} + \tau_{xy}\hat{j} + \tau_{xz}\hat{k}$$

$$\sigma_n = \bar{\sigma}_n \cdot \hat{n} = l^2\sigma_x + m^2\sigma_y + n^2\sigma_z + 2lm\tau_{xy} + 2ln\tau_{xz} + 2mn\tau_{yz}$$

mag of stress on plane

$$\bar{\sigma}_n = \sigma_n \hat{n} + \bar{\tau}$$

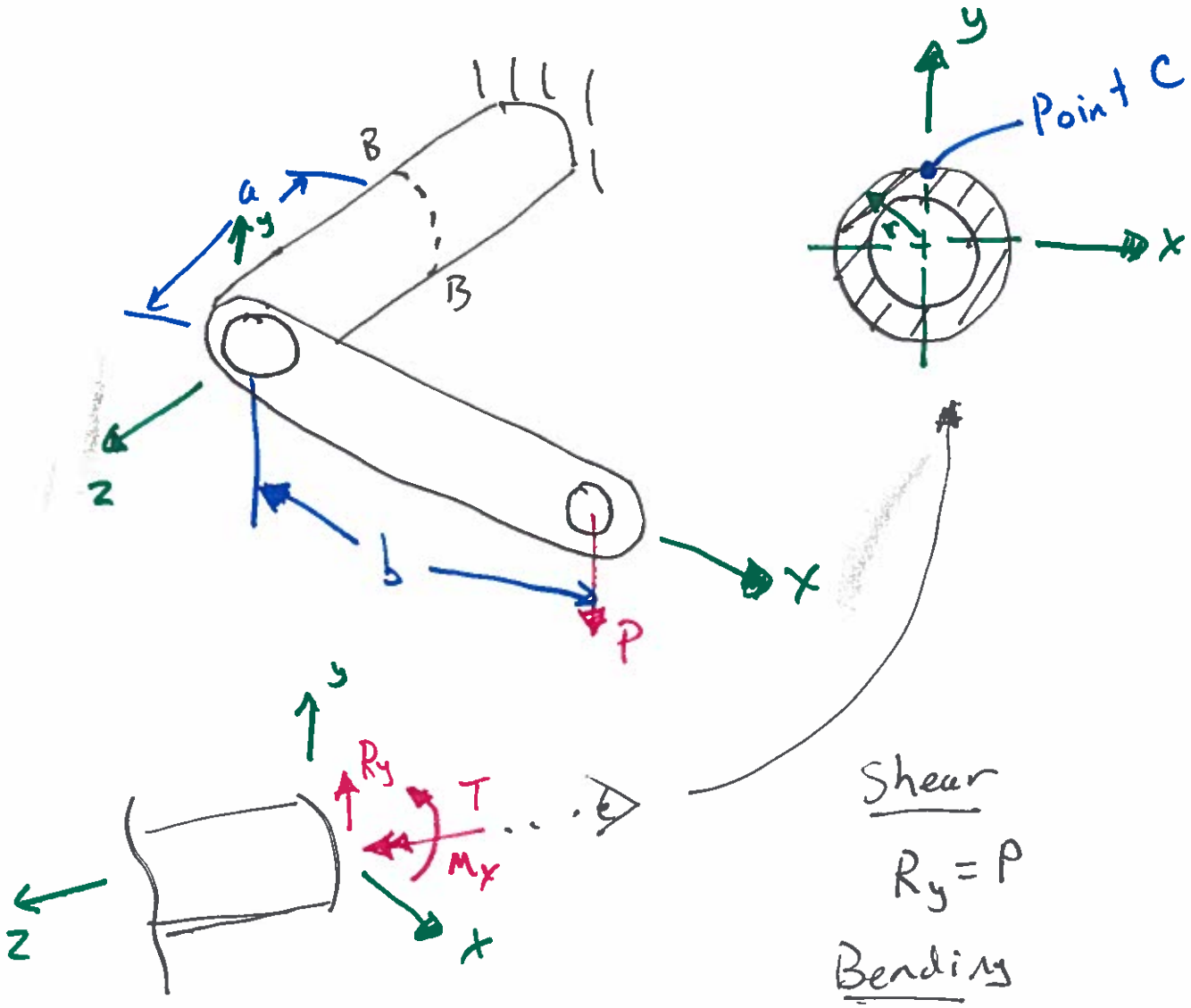


$$\hat{\tau} = (\hat{n} \times \hat{\sigma}_n) \times \hat{n}$$

$$|\bar{\sigma}_n|^2 = \sigma_n^2 + |\bar{\tau}|^2$$

} find mag & dir of $\bar{\tau}$

Example of finding multiaxial stress



Shear

$R_y = P$

Bending

$M_x = -Pa$

Torsion

$T = Pb$

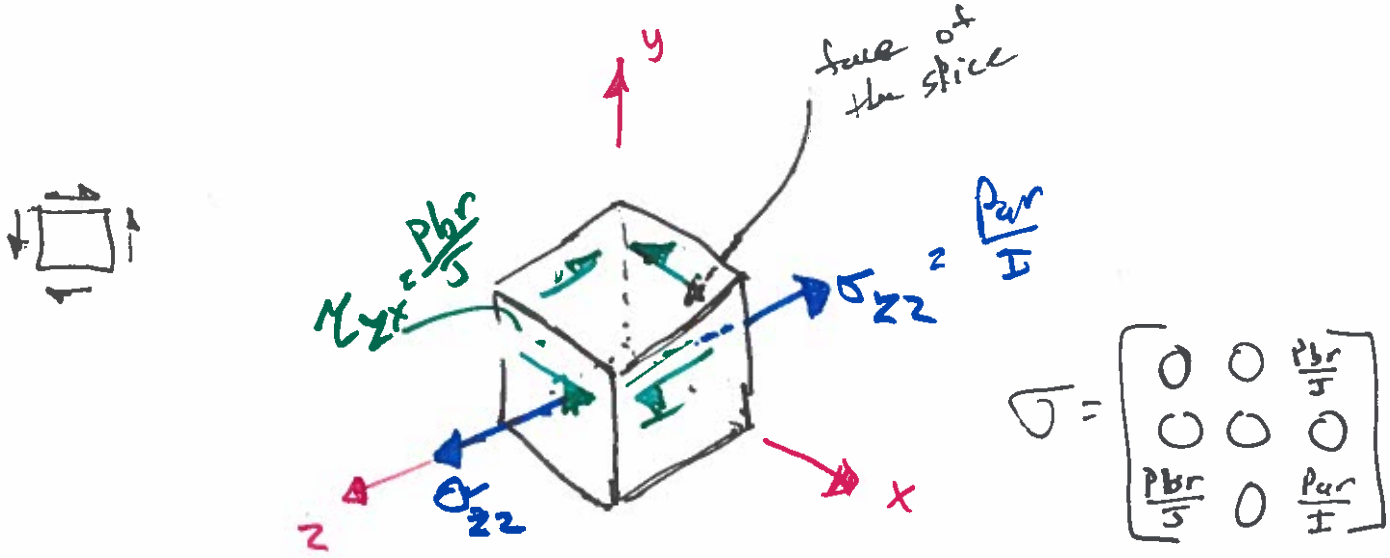
What is the stress tensor at point C?

I: second moment of area

A: area

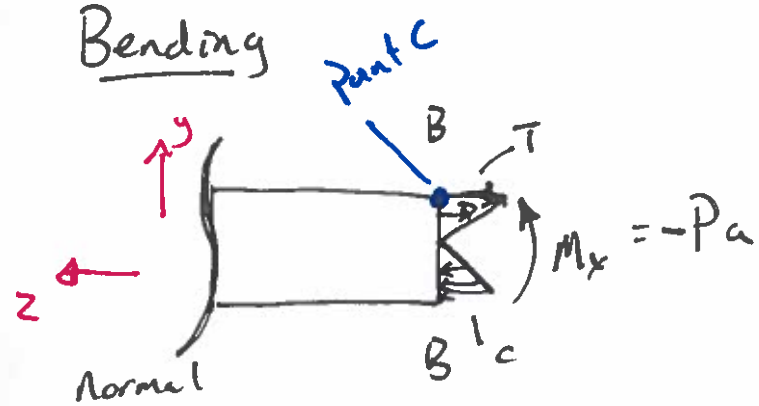
J: polar second moment of area

r: outer radius



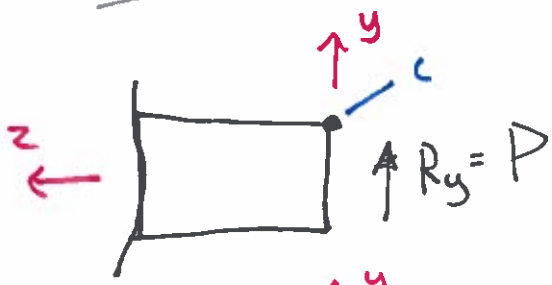
$$\sigma = \begin{bmatrix} 0 & 0 & \frac{Pbr}{J} \\ 0 & 0 & 0 \\ \frac{Pbr}{J} & 0 & \frac{Par}{I} \end{bmatrix}$$

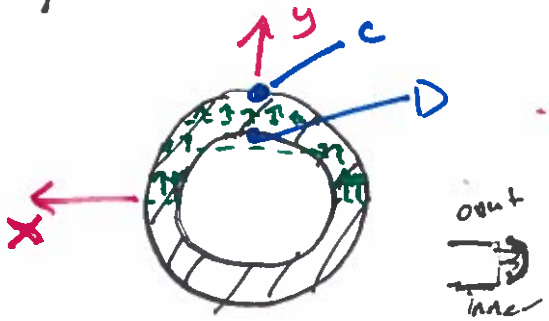
Bending



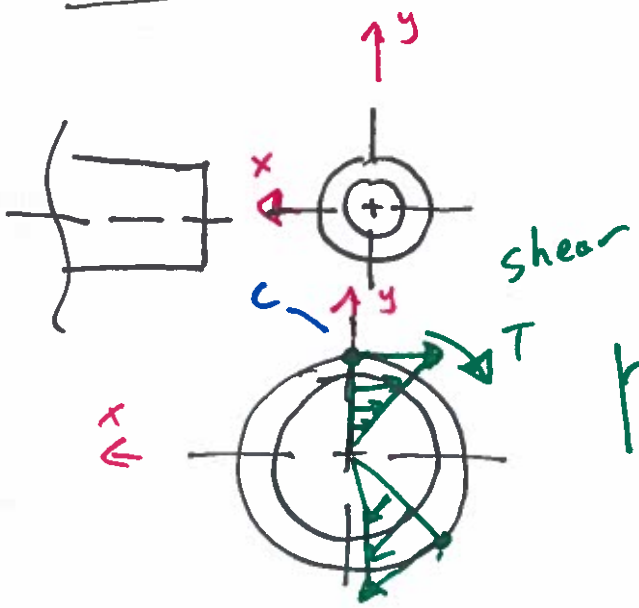
$$|\sigma_{zz}| = \frac{|M_x| r}{I} = \frac{Par}{I}$$

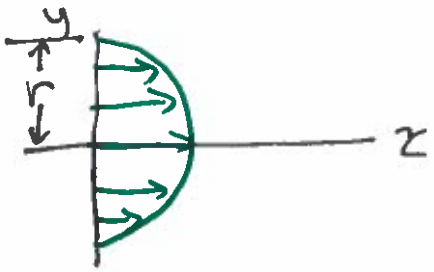
Transverse Shear





Torsional





$$|\tau_{zx}| = \frac{Pbr}{J} \qquad \tau_{zy} = 0$$