Chapters in book to review

1-1 to 1-16

3-1 to 3-14, 3-18, 3-19

4-1 to 4-6

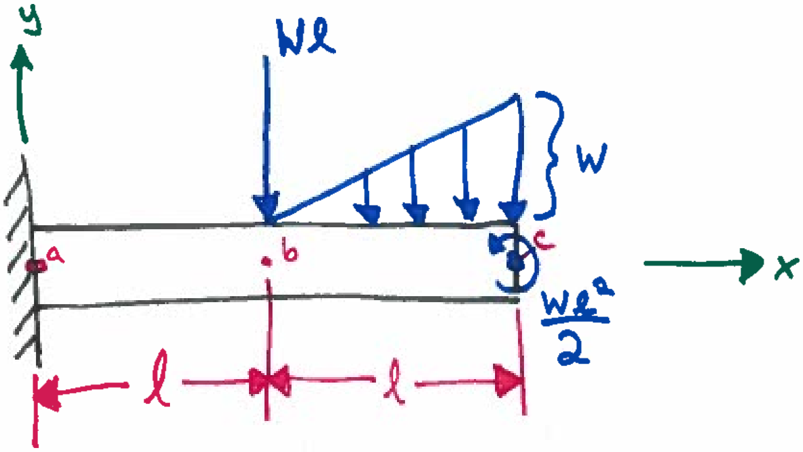
Main Topics

- Uncertainity, reliability, tolerances
- Design Factor and Factor of Safety
- State of stress/strain (prinicipal, max shear, concentrations)
- Deflection of basic elements (stiffness)
- Stress and Deflection of Beams (method of sections, { super-position, sing. functions })

Voted Topics

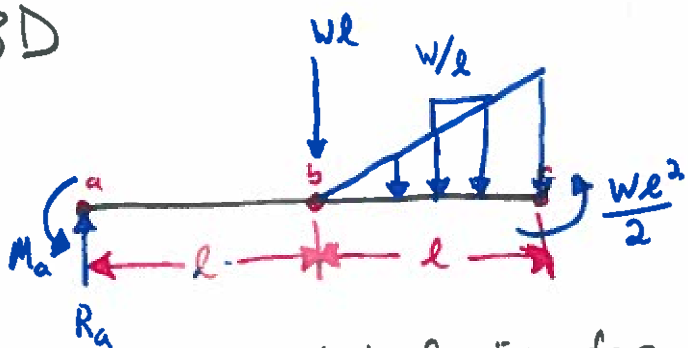
5 multiaxial stress (stress tensor)
3 sing. functions
3 transverse shear
2 strain
2 Mohr's circle
2 equations
⋮

Find the slope and deflection at the end ($x=2\ell$) using singularity functions.



Step 1: Draw FBD

FBD



Step 2: Write singularity function for loads, include all loads!

$$q(x) = R_a\langle x-0\rangle^{-1} - M_a\langle x-0\rangle^{-2} - W\ell\langle x-\ell\rangle^{-1} - \frac{W}{\ell}\langle x-\ell\rangle^{1}$$
$$- \frac{W\ell^2}{2}\langle x-2\ell\rangle^{-2}$$

Step 3: Integrate twice to find $V(x)$ and $M(x)$. Ignore constants of integration, because we will use the ℓ^+ trick to find reactions.

$$V(x) = R_a\langle x-0\rangle^{0} - M_a\langle x-0\rangle^{-1} - W\ell\langle x-\ell\rangle^{0} - \frac{W}{2\ell}\langle x-\ell\rangle^{2}$$
$$- \frac{W\ell^2}{2}\langle x-2\ell\rangle^{-1}$$

$$M(x) = R_a\langle x-0\rangle^{1} - M_a\langle x-0\rangle^{0} - W\ell\langle x-\ell\rangle^{1} - \frac{W}{6\ell}\langle x-\ell\rangle^{3}$$
$$- \frac{W\ell^2}{2}\langle x-2\ell\rangle^{0}$$

Step 4: Solve for reactions by substituting $x = 2\ell^+$, i.e. location just outside the beam.

$$V(2\ell^+) = R_a \langle 2\ell^+ - 0\rangle^0 - M_a \langle 2\ell^+ - 0\rangle^{-1} - w\ell \langle 2\ell^+ - \ell\rangle^0$$
$$- \frac{w}{2\ell}\langle 2\ell^+ - \ell\rangle^2 - \frac{w\ell^2}{2}\langle 2\ell^+ - 2\ell\rangle^{-1}$$

$$V(2\ell^+) = 0 = R_a(1) - M_a(0) - w\ell(1) - \frac{w}{2\ell}(\ell)^2$$
$$- \frac{w\ell^2}{2}(0)$$

$$0 = R_a - w\ell - \frac{w\ell}{2} \Rightarrow \boxed{R_a = \frac{3w\ell}{2}}$$

$$M(2\ell^+) = 0 = R_a \langle 2\ell^+ - 0\rangle^1 - M_a \langle 2\ell^+ - 0\rangle^0 - w\ell \langle 2\ell^+ - 0\rangle^1$$
$$- \frac{w}{6\ell}\langle 2\ell^+ - \ell\rangle^3 - \frac{w\ell^2}{2}\langle 2\ell^+ - 2\ell\rangle^0$$

$$0 = R_a(2\ell) - M_a(1) - w\ell(2\ell) - \frac{w}{6\ell}(\ell)^3 - \frac{w\ell^2}{2}(1)$$

$$M_a = 3w\ell^2 - 2w\ell^2 - \frac{w}{6}\ell^2 - \frac{w\ell^2}{2}$$

$$\boxed{M_a = \frac{w\ell^2}{3}}$$

Step 5: Substitute in reactions and drop singularities at $x = 2\ell$.

$$V(x) = \frac{3}{2}w\ell\langle x-0\rangle^0 - \frac{w\ell^2}{3}\langle x-0\rangle^{-1} - w\ell\langle x-\ell\rangle^0 - \frac{w}{2\ell}\langle x-\ell\rangle^2$$

$$M(x) = \frac{3}{2}w\ell\langle x-0\rangle^1 - \frac{w\ell^2}{3}\langle x-0\rangle^0 - w\ell\langle x-\ell\rangle^1 - \frac{w}{6\ell}\langle x-\ell\rangle^3$$

Step 6: Integrate twice more to get $\theta(x), V(x)$. Make sure to include EI and constants of integration.

$$EI\,\theta(x) = \frac{3}{4}w\ell\langle x-0\rangle^2 - \frac{w\ell^2}{3}\langle x-0\rangle^1 - \frac{w\ell}{2}\langle x-\ell\rangle^2 - \frac{w}{24\ell}\langle x-\ell\rangle^4 + C_\theta$$

$$EI\,y(x) = \frac{3}{12}w\ell\langle x-0\rangle^3 - \frac{w\ell^2}{6}\langle x-0\rangle^2 - \frac{w\ell}{6}\langle x-\ell\rangle^3 - \frac{w}{120\ell}\langle x-\ell\rangle^5 + C_\theta x + C_y$$

Step 7: Solve for constants of integration using the boundary conditions.

$$\theta(0) = 0 = \frac{3}{4}w\ell(0-0)^2 - \frac{w\ell^2}{3}(0-0)^1 - \frac{w\ell}{2}(0) - \frac{w}{24\ell}(0) + C_\theta$$

$$C_\theta = 0$$

$$y(0) = 0 = \frac{3}{12}w\ell(0-0)^3 - \frac{w\ell^2}{6}(0-0)^2 - \frac{w\ell}{6}(0) - \frac{w}{120\ell}(0) + C_\theta(0) + C_y$$

$$C_y = 0$$

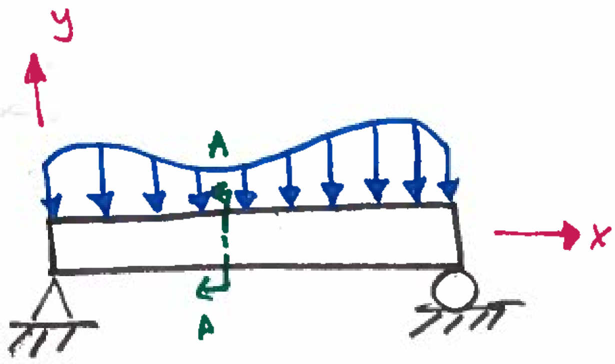
Step 8: Write expressions for $\theta(2\ell)$ and $y(2\ell)$

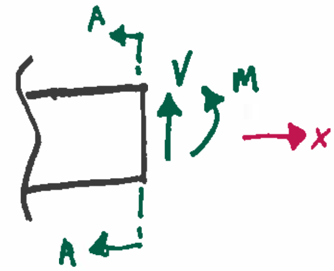
$$\theta(2\ell) = \frac{1}{EI}\left(\frac{3}{4}w\ell(2\ell)^2 - \frac{w\ell^2}{3}(2\ell)^1 - \frac{w\ell}{2}(2\ell-\ell)^2 - \frac{w}{24\ell}(2\ell-\ell)^4\right)$$

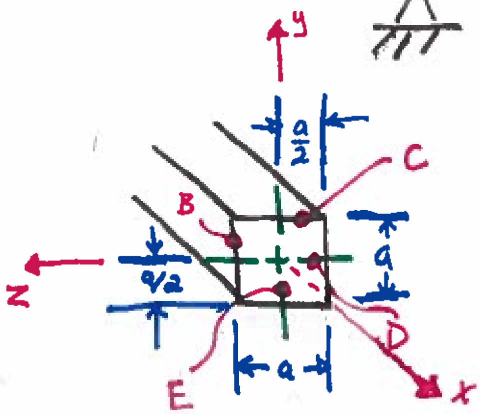
$$\theta(2\ell) = \frac{1}{EI}\left(3w\ell^3 - \frac{2}{3}w\ell^3 - \frac{1}{2}w\ell^3 - \frac{1}{24}w\ell^3\right) = \boxed{\frac{1}{EI}\frac{43}{24}w\ell^3}$$

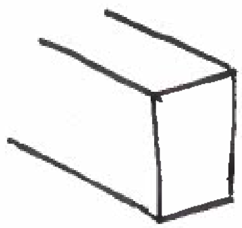
$$y(2\ell) = \frac{1}{EI}\left(\frac{1}{4}w\ell(2\ell)^3 - \frac{w\ell^2}{6}(2\ell)^2 - \frac{w\ell}{6}(2\ell-\ell)^3 - \frac{w}{120\ell}(2\ell-\ell)^5\right)$$

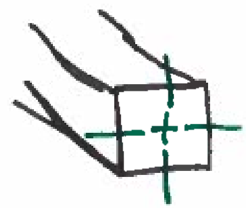
$$y(2\ell) = \frac{1}{EI}\left(2w\ell^4 - \frac{2}{3}w\ell^4 - \frac{w\ell^3}{6} - \frac{w\ell^4}{120}\right) = \boxed{\frac{1}{EI}\frac{139}{120}w\ell^4}$$









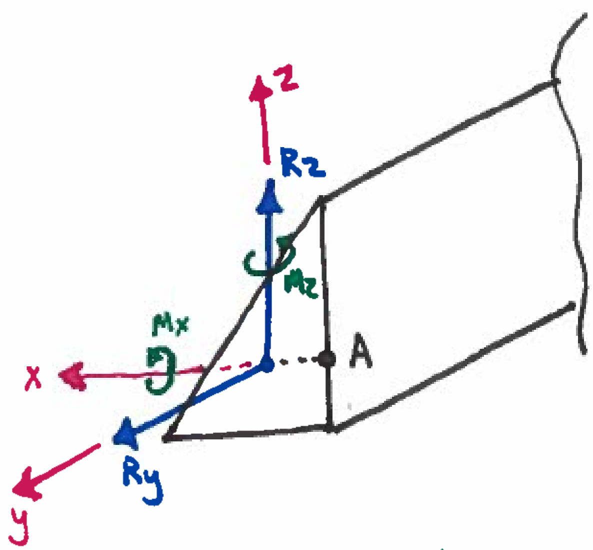


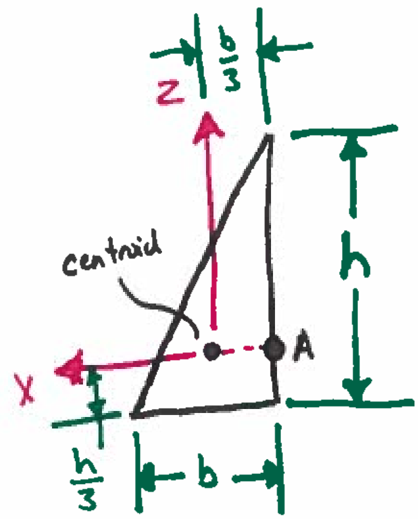
B: Compression and transverse shear

C: compression

D: transverse shear

E: tension and transverse shear





$$A = \frac{bh}{2}$$

$$I_x = \frac{bh^3}{36}$$

$$I_y = \frac{b^3 h}{36}$$

$R_y = -2\,\text{lb}$ $\quad$ $M_x = 10\ \text{in}\cdot\text{lb}$

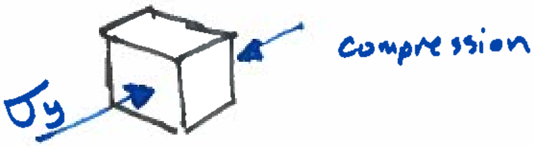
$R_z = 3\,\text{lb}$ $\quad$ $M_z = -5\ \text{in}\cdot\text{lb}$

$b = 1\ \text{in}$

$h = 2\ \text{in}$

Write the Cauchy Stress Tensor for point A and draw the correspond stress cube.

Axial



$$|\sigma_y| = \frac{|R_y|}{A} = \frac{|-2\text{ lb}|}{1\text{ in}^2} = 2\text{ psi}$$

Normal due to bending about x

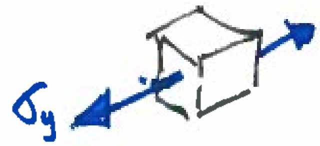
A is on the neutral axis

so $\sigma_y = 0$

Normal due to bending about z

at A there is tension

$$|\sigma_y| = \frac{|(5\text{ in·lb})|\left(\tfrac{1}{3}''\right)}{\dfrac{(2'')^3(2\text{ in})}{36}} = 3.75\text{ psi}$$



Shear

We have shear in the z direction on the y face and the is a moment about x, so we have to compute the transverse shear stress.

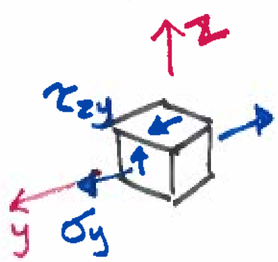
$$|\tau_{zy}| = \frac{|R_z|\,Q}{I t} \qquad t = \frac{2}{3}b \qquad I = \frac{bh^3}{36}$$

$$\bar{y}' = \frac{1}{3}\cdot\frac{2}{3}h = \frac{2}{9}h$$

$$A' = \frac{\frac{2}{3}b\cdot\frac{2}{3}h}{2} = \frac{2}{9}bh$$

$$|\tau_{zy}| = \frac{|3\text{ lb}|\,\bar{y}'A'}{I t} = \frac{|3\text{ lb}|\,\frac{2}{9}(2'')\,\frac{2}{9}(1'')(2'')}{\dfrac{(1'')(2'')^3}{36}\cdot\frac{2}{3}(1'')} = \frac{|3|\,\frac{16}{9}}{\frac{8}{36}\cdot\frac{2}{3}}$$

$$|\tau_{zy}| = 4\text{ psi}$$



$$\sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.75 & 4 \\ 0 & 4 & 0 \end{bmatrix}\text{ psi}$$