EME 150A LECTURE 20 FALL 2016 Monday, Nov 7, 2016

Compressive Loading

$$\sigma_x = \frac{P}{A} \Rightarrow \text{tensile failure}$$

σ_x can't be used to find the max stress and factor of safety wrt yield strength in compression.

In compression members may fail at a load lower that the compressive yield strength.

Have to consider:

Buckling!

Buckling occurs suddenly ⇒ very dangerous.

short columns → fail in compression $\sigma = \frac{P}{A}$

intermediate columns
long columns } may fail in buckling first.

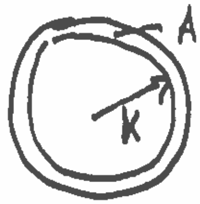
Categorized elements

$$S_r = \frac{l}{k}$$

l – length of column

k – radius of gyration

S_r – slenderness ratio

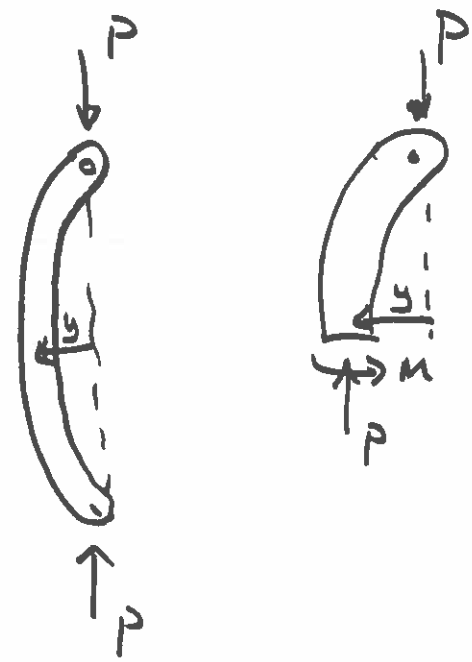


$$k = \sqrt{\frac{I_{min}}{A}}$$

$$I_{arbitrary} = I_{ring}$$

for $S_r \lesssim 10 \Rightarrow$ short beam

Long Columns



$$M = Py$$

small deflections

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = 0$$

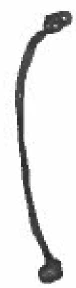
Second order ODE in y

$$y = \underline{A} \sin\left(\sqrt{\frac{P}{EI}}\, x\right) + \underline{B} \cos\left(\sqrt{\frac{P}{EI}}\, x\right)$$

boundary conditions

$y = 0$ @ $x = 0$

$y = 0$ @ $x = \ell$



$$\sin \sqrt{\frac{P}{EI}}\, \ell = 0$$

$$\sqrt{\frac{P}{EI}}\, \ell = n\pi \qquad n = 1, 2, 3, \ldots$$

First mode : static buckling behavior

$$\boxed{P_{cr} = \frac{\pi^2 EI}{\ell^2}}$$

→ Euler Column Formula

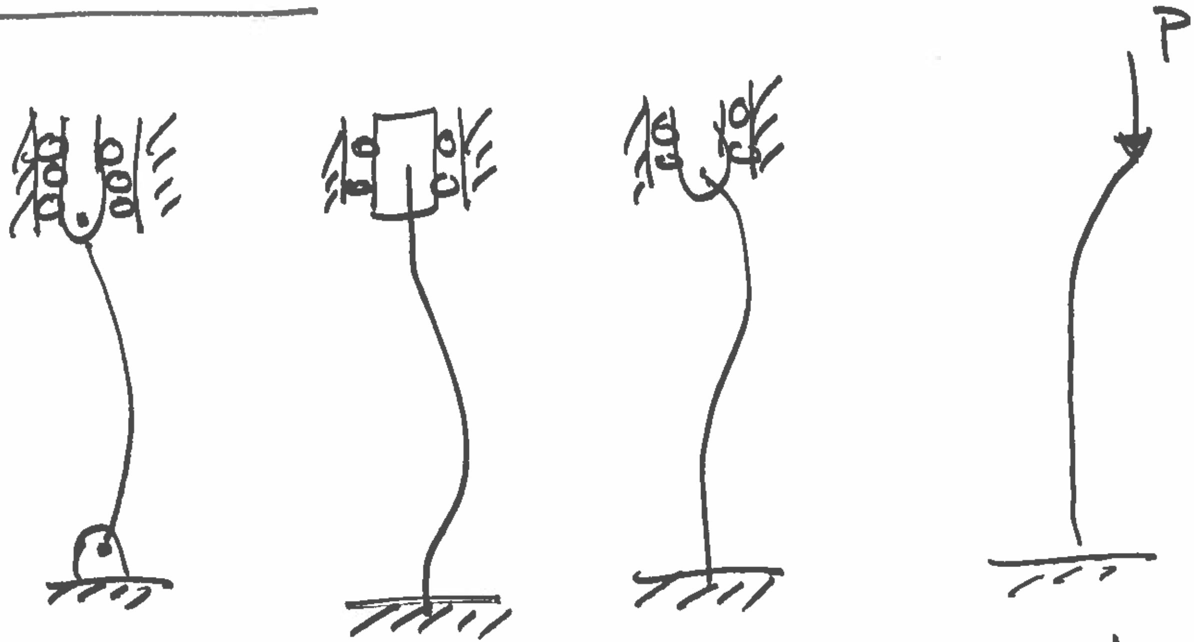
$$\boxed{\frac{P_{cr}}{A} = \frac{C\pi^2 E}{S_r^2}}$$

C: factor that accounts for different boundary conditions

S_r: slenderness ratio

$\frac{P_{cr}}{A}$: buckling strength

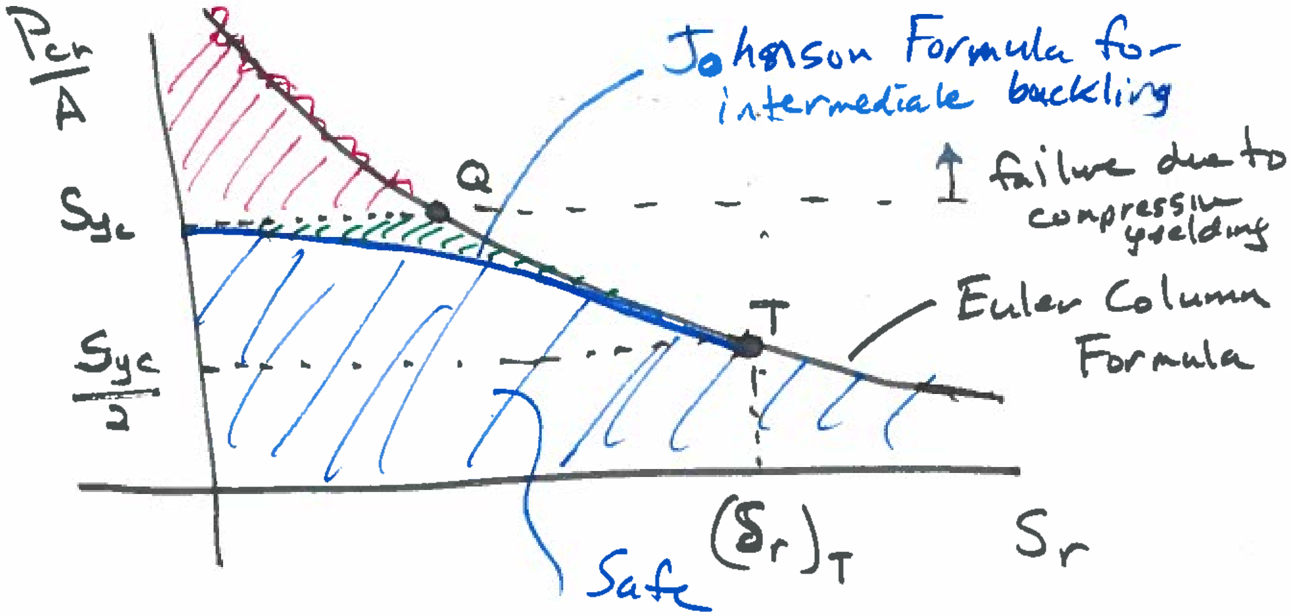
Table 4-2



$c = 1$ $\quad$ $c = 4$ $\quad$ $c = 2$ $\quad$ $c = \frac{1}{4}$

$c = 4$, $c = 2$ → $c = 1.2$ conservative, use in practice

$$\frac{P_{cr}}{A} = \frac{c\pi^2 E}{S_r^2}$$



Johnson Formula

$$\frac{P_{cr}}{A} = S_{yc} - \frac{1}{CE}\left(\frac{S_{yc} S_r}{2\pi}\right)$$ } blue curve

Example 4-17

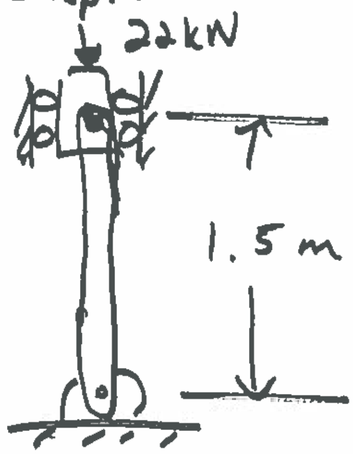
Find a diameter of a round column 1.5 m long that is carrying a compressive load of 22 kN.

Design Factor of 4

$S_y = 500$ MPa

$E = 207$ GPa

$C = 1$



step 2) $\left(\frac{l}{k}\right)_T = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = 90.4$

$S_r = \frac{l}{k} = \frac{l}{d/4} = 150$

Euler's colum formula

$\frac{P_{cr}}{A} = \frac{S_y}{2}$

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi}{4} d^2}}$

150 > 90.4 ⟹ long beam

step 1) $P_{cr} = n_d P = 4(22) = 88\,kN$

$d = \left(\frac{64 P_{cr} l^2}{\pi^3 CE}\right)^{\frac{1}{4}} = 37.48$ mm

$\frac{1}{4}$ → check!!

prefered size ⟹ **40mm**