EME 150A LECTURE 27 November 28, 2016

Finish up fatigue outline

High Cycle

$$S_f = a N^b \qquad N = \left(\frac{\sigma_{rev}}{a} \right)^{1/b}$$

$$a = \frac{(f S_{ut})^2}{S_e} \qquad b = -\frac{1}{3} \log\left(f S_{ut} / S_e \right)$$

Fluctuating Stresses

Find σ_m and σ_a (apply stress con. factors to both!)

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \frac{|\sigma_{max} - \sigma_{min}|}{2}$$

Apply failure criteria:

Finite life: mod. goodman line

$$\frac{1}{n} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}$$

Gerber is an alternative:

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

Check for yielding

$$\sigma_a + \sigma_m = S_y/n$$

$$\tau_a + \tau_m = \frac{0.577\, S_y}{n}$$

If you have pure torsion

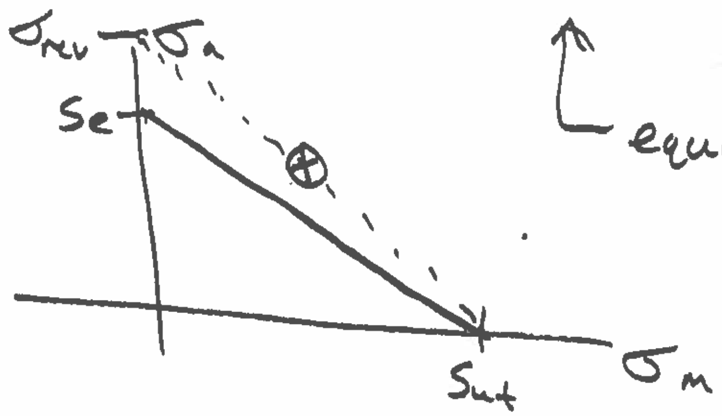
$\tau_m \geq 0$ $\quad$ $k_c = 0.59$ $\quad$ $S_{sy} = 0.556\, S_{yt}$

main parameter

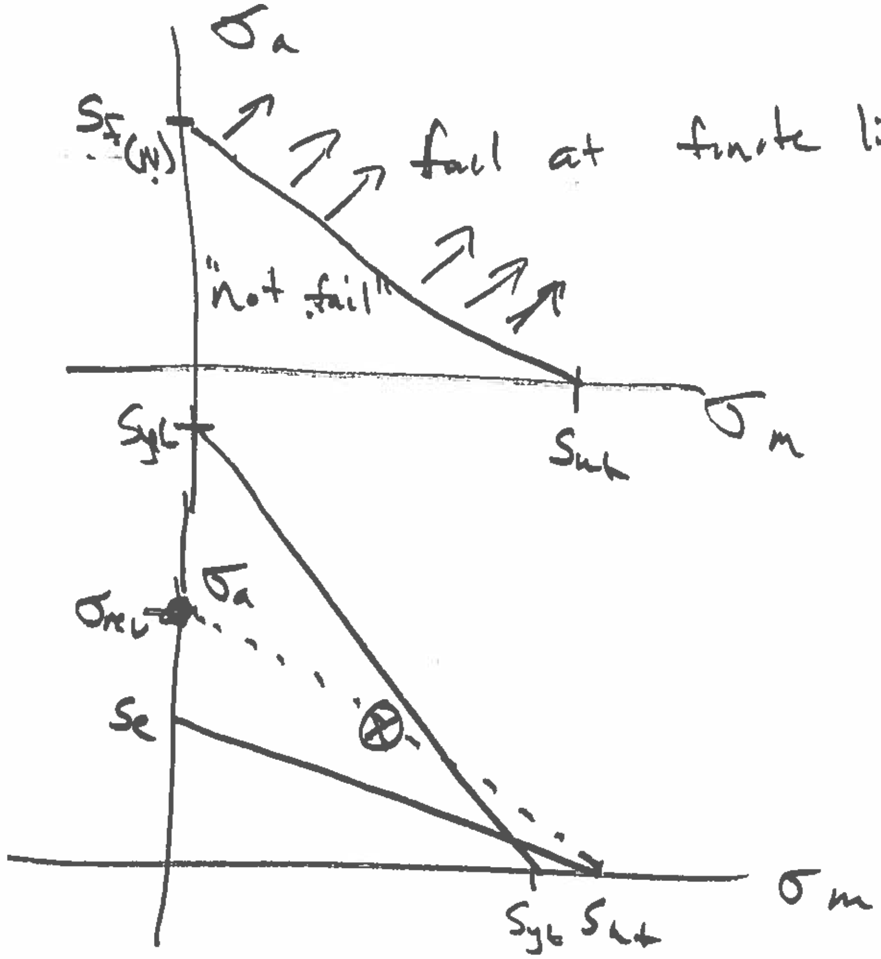
for fatigue $\quad S_{us} = 0.67\, S_{ut}$

Finite Life (fluctuating)

Mod-Goodman: $\quad \sigma_{rev} = \dfrac{\sigma_a}{1 - (\sigma_m / S_{ut})}$



equivalent fully reversed stress



$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b}$$

Compound Loading

Distortion Energy is used!

Most common case: bending + axial + torsion

$$\sigma_m' = \left\{ \left[(K_f)_{bend} (\sigma_m)_{bend} + (K_f)_{ax} (\sigma_m)_{ax} \right]^2 + 3\left[(K_{fs})_{tor} (\tau_m)_{tor} \right]^2 \right\}^{1/2}$$

$$\sigma'_a = \left\{ \left[(K_f)_{bend} (\sigma_a)_{bend} + (K_f)_{axial} \frac{(\sigma_a)_{ax}}{0.85} \right]^2 + 3\left[(K_{fs})_{tor} (\tau_a)_{tor} \right]^2 \right\}^{1/2}$$

K_c always be 1 when computing S_e

load factor

<u>reason</u>

The von mises stress includes the $k_c = 0.59$ for shear and we've includ $K_c = 0.85$ for axial above.

yield check: $\sigma'_a + \sigma'_m = \dfrac{S_y}{n}$

Conservatively

Cumulative Loading

Element is loaded:

σ_1' for n_1 cycles for which N_1 would produce failure.

σ_2' for n_2 cycles for which N_2 "

$\vdots$

σ_p' for n_p cycles for which N_p "

$$\underbrace{\sum \frac{n_i}{N_i} = C \quad \begin{array}{l} \text{if } C<1 \text{ failure will not occur} \\ \text{if } C \geq 1 \text{ failure will occur} \end{array}}_{\text{Miner's rule}}$$

How many cycles are remaing at a given stress?

If $C<1$, remaing life:

$$n_r = \left[1 - C_{current}\right] N_r$$

$$n_r = \left[C - \sum_{i=1}^{p} \frac{n_i}{N_i}\right] N_r$$

C taken as 1 in general

n_r remaining cycles at σ_r

Example A ~~[illegible]~~

Machined part is cycled $(\sigma_a)_1 = \pm 350$ MPa

$\sigma_m = 0$ for 5×10^3 cycles, then $(\sigma_a)_2 = \pm 260$ MPa

for 5×10^4 cycles, finally $(\sigma_a)_3 = \pm 225$ MPa.

How many cycles before failure?

given $S_{ut} = 530$ MPa, $f = 0.9$, $S_e = 210$ MPa

$$N = \left[\frac{\sigma_{rev}}{a}\right]^{1/b} \qquad a = \frac{(f S_{ut})^2}{S_e} \qquad b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right)$$

$N_1 = 13,550$ cycles

$N_2 = 165,600$ cycles

$N_3 = 559,400$ cycles

$$n_3 = \left[1 - \sum_{i=1}^{2} \frac{n_i}{N_i}\right] N_3$$

$$= \left[1 - \frac{5000}{13550} - \frac{50,000}{165,600}\right] 559400$$

$$\boxed{n_3 = 184,000 \text{ cycles}}$$

Fluctuating Stress Example

1.5" dia bar machined from CD 1050 steel has fluctuating load from 0-16,000 lbs (tensile). Assume that $K_f = 1.85$

Find: n_{inf} using mod. Goodman line

↑ factor of safety

Solution:

$S_e' \Rightarrow S_{ut} = 100$ kpsi , $S_y = 84$ kpsi

$S_e' = 0.5\, S_{ut}$ since $S_{ut} < 200$ kpsi

$S_e' = 50$ kpsi

$S_e = k_a k_b k_c S_e'$

$k_a = a S_{ut}^{\,b}$ get a & b from Table 6-2

$= 2.7(100)^{-.265}$

$K_a = 0.797$

$K_b = 1$ eqn 6-21

$K_c = 0.85$ eqn 6-26

$S_e = 33.87$ kpsi

$$(\sigma_m)_0 = \frac{F_m}{A} = \frac{8000}{\frac{\pi(1.5)^2}{4}} = 4.53 \text{ Kpsi}$$

$$(\sigma_a)_0 = \sigma_m = 4.53 \text{ Kpsi}$$

$$\sigma_a = K_f (\sigma_a)_0 = 8.38 \text{ Kpsi}$$

<u>Goodmam</u>

$$n_{inf} = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} \qquad n_{mf} = 3.02$$ ✓

<u>Yield</u>

$$n_{yield} = \frac{S_y}{\sigma_a + \sigma_m} \Rightarrow n_{yield} = 5.01$$ ✓