Equations of Motion

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$$\sum \bar{F} = \frac{d\bar{p}}{dt}$$ linear momentum

$$\sum \bar{T} = \frac{d\bar{H}}{dt}$$ angular momentum

Newton's Second Law

$\bar{p} = m\bar{v}$

$\bar{H} = I\,\bar{\omega}$

I: Inertia tensor (matrix)

$\bar{\omega}$: ang. vel

These 2nd order ordinary differential equations in time.

Essential elements to vibrating systems

1. Inertia of the oscillating mass (store energy, kinetic)
2. Restoring forces (elastic, gravity) [store energy, potential)
3. Dissapative mechanism (Energy loss)

of Degrees of Freedom

the number of independent coordinates to describe the motion of the system

positions, angles, etc

Procedure

1. Mathematical Model of reality
2. Derive the EoMs
3. Seek the solutions to the EoMs
4. Interpret the results.

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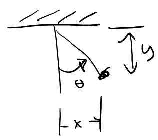
Concepts:

- rigid bodies and particles (lumped masses)
  non flexible
- DoF
- reference frames (we need inertial frame to form the EoMs wrt)
  no acceleration
- coordinates (positions and angles that describe configuration)
- Generalized Coordinates: a minimum set of coordinates that uniquely describe the configuration



θ or ~~x, y~~
G.C.
are not G.C.s

- Speeds: $\frac{d}{dt}$ of G.C.s.

Derivation of EoMs

(I) Direct Method
(FBD + applied Newton's Laws to form F=m)

(II) Indirect Methods
(Lagrange, Hamilton, Kane's, etc)

Lagrange's Method

"Energy Method"

Kinetic energy:

$$T = \underbrace{\frac{1}{2}\sum_{i=1}^{N} m_i v_i^2}_{\text{linear motion term}} + \overbrace{\frac{1}{2}\sum_{i=1}^{M} I_i \omega_i^2}^{\text{planar rotation}}$$

K.E.

mass of particle

mag. of velocity of the particle

moment of inertia about centroid

mag. ang. vel. of RB

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only look at the linear motion term

$$\frac{\partial T}{\partial v_i} = \sum \underbrace{m_i v_i}_{\text{linear momentum}}$$

$$T = \frac{1}{2} \sum m_i v_i^2$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial v_i}\right) = \sum m_i \dot{v}_i = \underbrace{\sum m_i a_i}_{F=ma}$$

Conservative Forces (restoring force spring, gravity)

$$F_i = -\frac{\partial U}{\partial x_i}$$ ← potential energy

x_i ← generalized coordinate

$$\underbrace{\frac{d}{dt}\left(\frac{\partial T}{\partial v_i}\right) = -\frac{\partial U}{\partial x_i}}_{F=ma}$$

Defined the Lagrangian

$$L = T - U$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial v_i}\right) = \frac{\partial L}{\partial x_i}}$$

$$\frac{\partial T}{\partial x_i} = 0, \quad \frac{\partial U}{\partial v_i} = 0$$

Lagrange Equations (of the second kind)