ENG122-L11-01

Wednesday, November 1, 2017 1:03 PM

Kinetic Energy, T

(storage of energy due to motion)

(planar)

$T_{linear} = \frac{1}{2} m v^2$

mass of particle or mass center of RB

mag of its vel

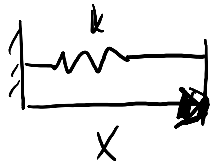
$T_{rot} = \frac{1}{2} I_c \omega^2$

centroidal moment of inertia of a RB.

mag. of the RB.'s angular velocity

Potential Energy, U

conservative forces, storage (static)



$U = \frac{1}{2} k x^2$

$\frac{\partial U}{\partial x} = kx = F$

$U = mgh$

$I_c : kg\,m^2$

$\omega = \frac{rad}{s}$

Energy dissaption

non-conservative forces

cause energy loss

viscous damping: $\underset{N}{F_{damping}} = \overset{kg/s}{c}\,\overset{m/s}{v}$

Rayliegh's dissepation Function:

$R = \frac{1}{2} \sum_{i=1}^{N} c_i v_i^2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial v_i}\right) - \frac{\partial L}{\partial x_i} = Q_i$

$Q_i = -\frac{\partial R}{\partial v_i}$

Lagrange's Equation

$$\left.\frac{d}{dt}\left(\frac{\partial L}{\partial v_i}\right) - \frac{\partial L}{\partial x_i} = 0 \right\}$$

only valid for systems with con. forces

$L = T - U$

T : Joules

U : Joules

$F_s = k x$

$N = \frac{N}{m}\, m$

$kg/s \; \frac{m^2}{s^2} = N \; m/s$

$F v = P$

ENG122-L11-02

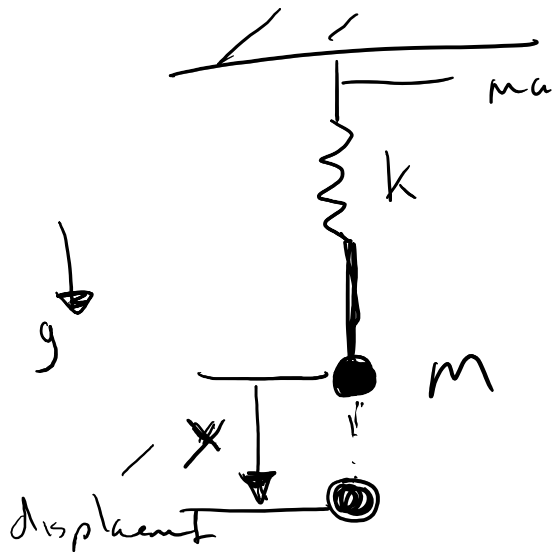
Wednesday, November 1, 2017 4:35 PM

Modeling

1. What are trying to answer? Choose the simplest model that answers our question correctly. (Making assumptions)
2. Identify important elements in the system (R.Bs, particles, force generators, etc)
3. Draw a Free Body Diagram
4. Adding Forces/Torque to FBD and identify and adding a <u>minimal</u> set generlazted coordinates.
5. Write expressions for T, U, (Q)
6. Write L and Lagrange's Equation
7. Take derivatives
8. optionally linearize the equations (simplifies system, only do if fits assumptions)
9. Seek solutions to diff. eqs. (analytically or numerically) gives trajectories of the G.Cs
10. Interpret the simulation results

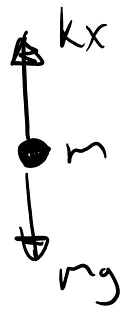
ENG122-L11-03

Wednesday, November 1, 2017 4:49 PM



displacement away from equilibrium point

$F = kx$



$$\frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) - \frac{\partial L}{\partial x} = 0$$

$$m\dot{v} + kx - mg = 0$$

$$m\dot{v} = mg - kx$$

$$ma = \Sigma F$$

$$T = \frac{1}{2}mv^2$$

$$U_1 = mgh = mg(-x)$$

$$U_2 = \frac{1}{2}kx^2$$

$$U = U_1 + U_2$$

$$L = \frac{1}{2}mv^2 - \frac{1}{2}kx^2 + mgx$$

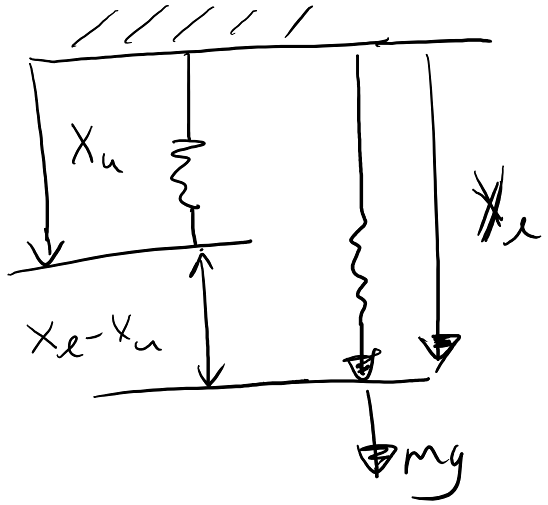
$$\frac{\partial L}{\partial v} = mv \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) = m\dot{v}$$

$$\frac{\partial L}{\partial x} = -kx + mg$$

Canonical form

$$\underbrace{m\dot{v} + kx}_{\text{linear system}} = mg$$

2nd order ODE linear



$$mg = k(x_e - x_u)$$

$$x_e = \frac{mg}{k} + x_u$$

static displacement due to mass

$$\overset{0}{\cancel{m\dot{v}}} + kx = mg$$

$$x = \frac{mg}{k}$$