$M\ddot{\bar{q}} + C\dot{\bar{q}} + K\bar{q} = F$

↓ state space form (first order form)

$\dot{\bar{x}} = A\bar{x} + Bu$ $\quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$ $\quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$

state matrix contains the dynamics

$\ddot{\bar{q}} + \underbrace{M^{-1}C\dot{\bar{q}} + M^{-1}K\bar{q}} = M^{-1}F$

$\bar{x} = \begin{bmatrix} \bar{q} \\ \dot{\bar{q}} \end{bmatrix}$ $\quad \ddot{\bar{q}} = -M^{-1}C\dot{\bar{q}} - M^{-1}K\bar{q} + M^{-1}F$

$u = \dot{q}$ $\quad \bar{X} = \begin{bmatrix} q \\ u \end{bmatrix}$ $\quad \dot{\bar{x}} = \begin{bmatrix} \dot{q} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}\begin{bmatrix} \bar{q} \\ \bar{u} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}[\bar{F}]$

non linear first order

$\dot{\bar{X}} = f(\bar{x}, t)$

SDOF

$m\ddot{x} + c\dot{x} + kx = f$ canonical form

$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{f}{m}$ mass normalized canonical form

$\omega_n = \sqrt{\frac{k}{m}}$ $\quad \zeta$: damping ratio

$\zeta = \frac{c}{c_{cr}}$ $\quad 0 < \zeta$

$c_{cr} = 2m\omega_n = 2\sqrt{km}$

if $\zeta > 1$: overdamped

if $\zeta = 1$: critical damping

if $0 \leq \zeta < 1$: underdamped

underdamped $0 < \zeta < 1$

$x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d + \phi)$

eigenvalues: complex conjugate pair

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

↳ damped nat freq

$e^{ix} = \cos x + i\sin x$ Euler formula

$e^{i\pi} = \cos\pi + i\sin\pi = -1$

$\tan\phi = \frac{x_0\omega_d}{v_0 + \zeta\omega_n x_0}$ $\quad A = \sqrt{\frac{(v_0 + \zeta\omega_n x_0)^2 + (x_0\omega_d)^2}{\omega_d}}$

$\lambda_1 = -\zeta\omega_n - \omega_n\sqrt{1-\zeta^2}\,i$

$\lambda_2 = -\zeta\omega_n + \omega_n\sqrt{1-\zeta^2}\,i$

$x(t) = Ce^{\lambda t}$

overdamped $\zeta > 1$ pair of distinct real eval

$\lambda_1 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}$

$\lambda_2 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}$

$x(t) = a_1 e^{\lambda_1 t} +$

$a_1 = \frac{-v_0 + (-\zeta + \sqrt{\zeta^2-1})\omega_n x_0}{2\omega_n\sqrt{\zeta^2-1}}$ $\quad a_2 = \frac{v_0 + (\zeta +}{2\omega_n\sqrt{\zeta^2-}}$

critically damped $\zeta = 1$

$x(t) = (a_1 + a_2 t)e^{-\omega_n t}$

$a_1 = x_0$ $\quad a_2 = v_0 + \omega_n x_0$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}$$

$$eig(A) = 0$$

s

$$a_2 e^{\lambda_2 t}$$

$$\frac{\sqrt{\zeta^2 - 1})\omega_n x_0}{1}$$

$$\lambda_1 = \lambda_2 = -\omega_n$$