MAE223-L5-01

Wednesday, October 11, 2017 8:45 AM

$$^A\bar{v}^{P_1} = \dot{q}_1 \hat{b}_x + \dot{q}_2 \hat{b}_y - \omega q_1 \hat{b}_z$$

how do I express in the (E) frame?

$$\boxed{^A\bar{v}^{P_1} = \dot{q}_1 (c_3 \hat{e}_x - s_3 \hat{e}_y) + \dot{q}_2 (s_3 \hat{e}_x + c_3 \hat{e}_y) - \omega q_1 \hat{e}_z}$$

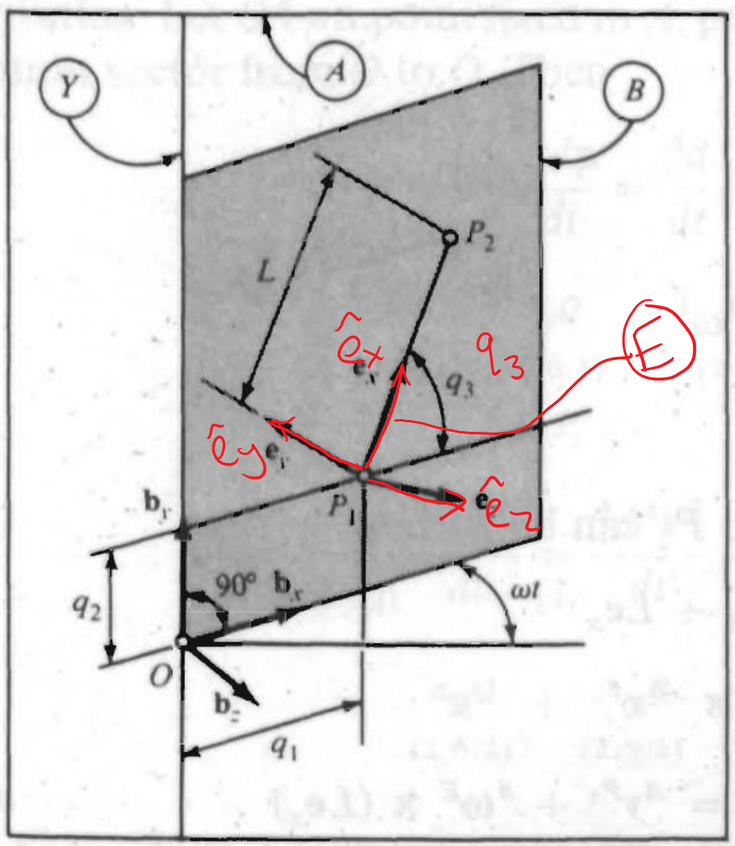


Figure 2.6.1

$$\hat{b}_x = c_3 \hat{e}_x - s_3 \hat{e}_y$$

$$\hat{b}_y = s_3 \hat{e}_x + c_3 \hat{e}_y$$

$$\hat{b}_z = \hat{e}_z$$

$$^A\bar{v}^{P_2} = \frac{^A d\bar{P}_2}{dt} = \frac{^A d\bar{P}_1}{dt} + \frac{^A d(\bar{P}_2 - \bar{P}_1)}{dt}$$

$$= {^A\bar{v}^{P_1}} + \frac{^A d L\hat{e}_x}{dt}$$

$$= {^A\bar{v}^{P_1}} + \cancel{\frac{^E d L\hat{e}_x}{dt}}^{\,0} + {^A\bar{\omega}^E} \times L\hat{e}_x$$

$$^A\bar{v}^{P_2} = {^A\bar{v}^{P_1}} + {^A\bar{\omega}^E} \times L\hat{e}_x$$

$$^A\bar{\omega}^E = {^A\bar{\omega}^B} + {^B\bar{\omega}^E} = \omega \hat{b}_y + \dot{q}_3 \hat{e}_z$$

$$^A\bar{\omega}^E \times L\hat{e}_x = (\omega \hat{b}_y + \dot{q}_3 \hat{e}_z) \times L\hat{e}_x = -\omega L \hat{e}_z + \dot{q}_3 L \hat{e}_y$$

Two points on the same rigid body.
How are the velocities related?

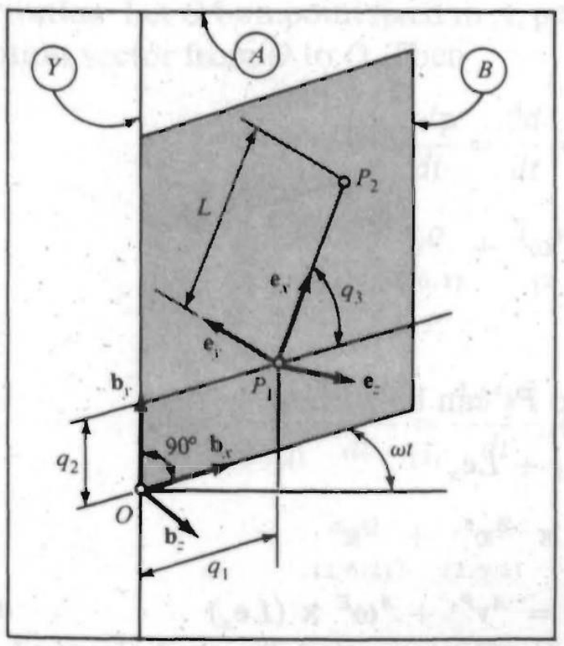
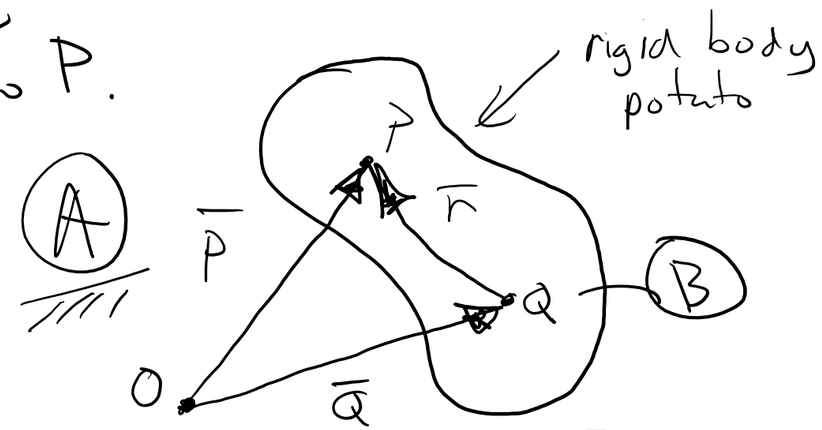


Figure 2.6.1

What RF are P_1 and P_2 fixed in relative to each other?

Given RF A and 2 points fixed in body B with angular velocity ${}^A\bar{\omega}^B$, with $\bar{p}, \bar{q}$ the position vectors from O (fixed in A) to P, Q, resp. and $\bar{p} = \bar{q} + \bar{r}$ so $\bar{r}$ is a vector from Q to P.



Velocity two point theorem

$$\boxed{{}^A\bar{V}^P = {}^A\bar{V}^Q + {}^A\bar{\omega}^B \times \bar{r}}$$

fixed in B

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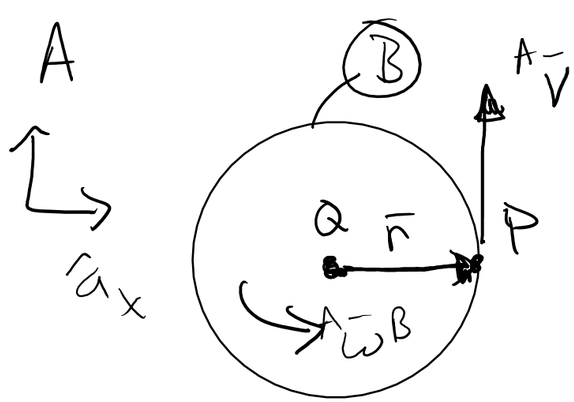
$$ {}^A\bar{a}^P = \frac{{}^A d\,{}^A\bar{v}^P}{dt} = \frac{{}^A d\,{}^A\bar{v}^Q}{dt} + \frac{{}^A d\left({}^A\bar{\omega}^B \times \bar{r}\right)}{dt} $$

$$ {}^A\bar{a}^P = {}^A\bar{a}^Q + \frac{{}^A d\,{}^A\bar{\omega}^B}{dt} \times \bar{r} + {}^A\bar{\omega}^B \times \frac{{}^A d\bar{r}}{dt} $$

$$ \boxed{{}^A\bar{a}^P = {}^A\bar{a}^Q + \underbrace{{}^A\bar{\alpha}^B \times \bar{r}}_{\text{tangential acceleration}} + \underbrace{{}^A\bar{\omega}^B \times \left({}^A\bar{\omega}^B \times \bar{r}\right)}_{\text{radial (centripetal) acceleration}}} $$

acceleration two point theorem

ex



${}^A\bar{v}^P = {}^A\bar{\omega}^B \times \bar{r}$

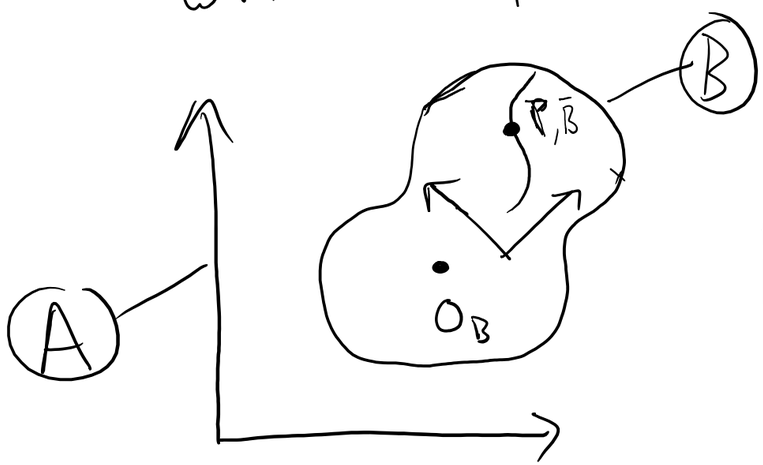
${}^A\bar{v}^Q = 0 = {}^A\bar{a}^Q$

$$ {}^A\bar{a}^P = \cancel{{}^A\bar{a}^Q}^{\,0} + \cancel{{}^A\bar{\alpha}^B}^{\,0} \times \bar{r} + \underbrace{{}^A\bar{\omega}^B \times \left({}^A\bar{\omega}^B \times \bar{r}\right)}_{-a_x} $$

if ω is constant then

and $\longrightarrow -r\omega^2 \hat{a}_x$

2.8 One point P moving on RB B (or RF B) while B moves in RF A. Let $\bar{B}$ be the point fixed in B where P is at this instance in time ($\bar{B}$ coincides with P presently)



$$\left.\begin{array}{l} {}^{A}\bar{\omega}^{B} \neq 0 \\ {}^{A}V^{O_B} \neq 0 \end{array}\right\} \text{B moving in A}$$

$$\boxed{{}^{A}\bar{V}^{P} = {}^{B}\bar{V}^{P} + {}^{A}\bar{V}^{\bar{B}}}$$