Partial Velocities and Partial Angular Velocities

$q_1, \ldots, q_n$ GC's $\qquad u_1, \ldots, u_n$ G.S's

Have lots of rigid bodies and points in system whose motion is of interest. All velocities and angular velocities in this system can be expressed uniquely as functions of the u_i's.

$$\bar{\omega} = \sum_{r=1}^{n} \bar{\omega}_r u_r + \bar{\omega}_t$$

$\bar{\omega}_r$: rth partial angular velocity; u_r: rth G.S.; $\bar{\omega}_t$: remainder vector

$$\bar{V} = \sum_{r=1}^{n} \bar{V}_r u_r + \bar{V}_t$$

$\bar{V}_r$: rth partial velocity

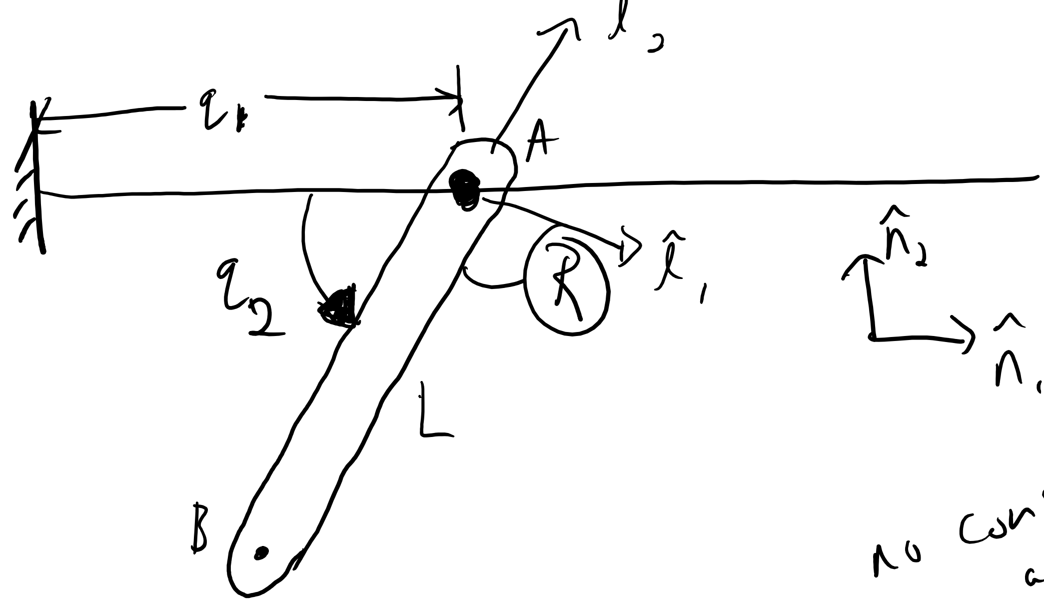
$\bar{\omega}_r, \bar{V}_r, \bar{\omega}_t, \bar{V}_t$ are functions of $q_1, \ldots, q_n, t$

Note that

$$\bar{\omega}_r = \frac{\partial \bar{\omega}}{\partial u_r}$$

$$\bar{V}_r = \frac{\partial \bar{V}}{\partial u_r}$$

ex: Calculation of partial velocities



no config con. as defined

2 G.C.s : q_1, q_2 $n=2$

DOF: 2 $p=n-m$

no motion constraints!

$$\bar{V} = \sum \underbrace{\bar{V}_r u_r} + \bar{V}_t$$

$$\boxed{{}^N\bar{V}^A_1 = \hat{n}_1} \qquad \boxed{{}^N\bar{V}^A_2 = 0}$$

$$\bar{V} = u_1 + \underbrace{0 \cdot u_2} + 0$$

$u_1 = \dot{q}_1$ ← I define!

$$\boxed{{}^N V^A} = \dot{q}_1 \hat{n}_1 = \boxed{u_1 \hat{n}_1}$$

$$^N V^B = {}^N V^A + {}^A\bar{\omega}^B \times \bar{r}^{B/A}$$

$$= u_1 \hat{n}_1 + \dot{q}_2 \hat{n}_3 \times -L\hat{l}_2$$

$$= u_1 \hat{n}_1 + \dot{q}_2 \hat{n}_3 \times (-Lc_2 \hat{n}_1 - Ls_2 \hat{n}_2)$$

$$^N V^B = (u_1 + L\dot{q}_2 s_2)\hat{n}_1 - L\dot{q}_2 c_2 \hat{n}_2$$

$u_2 = \dot{q}_2$

$$\boxed{^N V^B = (u_1 + L u_2 s_2)\hat{n}_1 - L u_2 c_2 \hat{n}_2}$$

$$\boxed{^A\bar{\omega}^B = u_2 \hat{n}_3}$$

* eliminate all $\dot{q}_i$ from vels.*

$$^N\bar{V}^B_1 = ? = \hat{n}_1 \qquad ^N\bar{V}^B_2 = ? = \hat{n}_1 L s_2 - \hat{n}_2 L c_2$$

$$^A\bar{\omega}^B_1 = ? = 0 \qquad ^A\bar{\omega}^B_2 = ? = \hat{n}_3$$

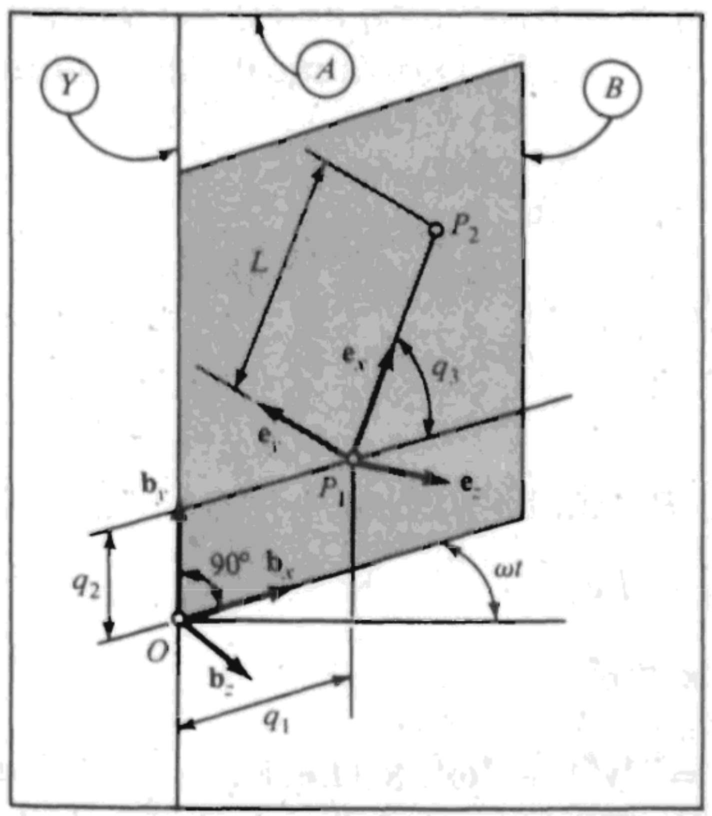
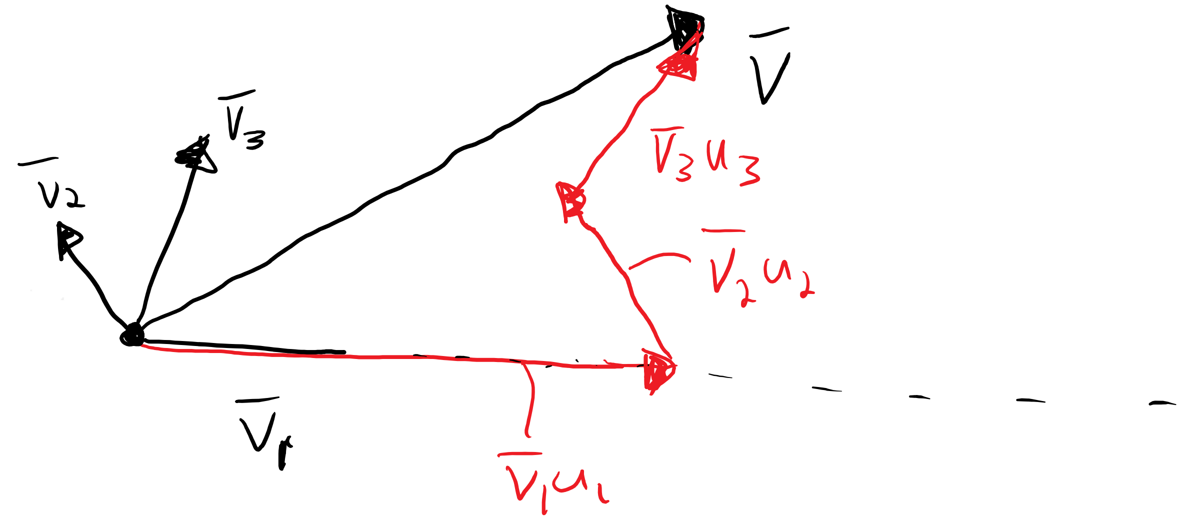


Figure 2.6.1



partial velocities represent the sensitivity of $\bar{v}$ or $\bar{\omega}$ to the chosen C.S.s.

(1) ${}^A\bar{V}_1^{P_1} = \hat{b}_x$

${}^A\bar{V}_2^{P_1} = \hat{b}_y$

${}^A\bar{V}_3^{P_1} = 0$

${}^A\bar{V}_t^{P_1} = -\omega q_1 \hat{b}_z$

(2) ${}^A\bar{V}_1^{P_1} = \hat{e}_x$

${}^A\bar{V}_2^{P_1} = \hat{e}_y$

${}^A\bar{V}_3^{P_1} = 0$

${}^A\bar{V}_t^{P_1} = -\omega q_1 \hat{e}_z$

$${}^A\bar{V}^{P_1} = \dot{q}_1 \hat{b}_x + \dot{q}_2 \hat{b}_y - \omega q_1 \hat{b}_z$$

$${}^A\bar{V}^{P_1} = (\dot{q}_1 c_3 + \dot{q}_2 s_3)\hat{e}_x + (-\dot{q}_1 s_3 + \dot{q}_2 c_3)\hat{e}_y - \omega q_1 \hat{e}_z$$

$${}^A\bar{V}^{P_1} = [\dot{q}_1 \cos(\omega t) - \omega q_1 \sin(\omega t)]\hat{a}_x + \dot{q}_2 \hat{a}_y - [\dot{q}_1 \sin(\omega t) + \omega q_1 \cos(\omega t)]\hat{a}_z$$

(1) $u_1 = \dot{q}_1, \; u_2 = \dot{q}_2, \; u_3 = \dot{q}_3$

(2) $u_1 = \dot{q}_1 c_3 + \dot{q}_2 s_3, \; u_2 = \dot{q}_2 c_3 - \dot{q}_1 s_3, \; u_3 = \dot{q}_3$

Chapter 1

- vectors (measurements and components)
- reference frames (mutually perpendicular unit vectors)
- derivatives of vectors in different reference frames
- Diff. sums, products:
- Total derivatives (chain rule)

$$\frac{{}^A\partial \bar{v}}{\partial q} = \sum_{i=1}^{3} \frac{\partial v_i}{\partial q}\hat{a}_i \quad \text{where} \quad \bar{V} = v_1\hat{a}_1 + v_2\hat{a}_2 + v_3\hat{a}_3$$

$$\frac{{}^A d\bar{v}}{dt} = \sum_{i=1}^{3} \frac{{}^A\partial \bar{v}}{\partial q_r}\dot{q}_r + \frac{\partial \bar{v}}{\partial t}$$

Kinematics

$${}^A\bar{\omega}^B \triangleq \hat{b}_1 \frac{{}^A d\hat{b}_2}{dt}\cdot\hat{b}_3 + \hat{b}_2 \frac{{}^A d\hat{b}_3}{dt}\cdot\hat{b}_1 + \hat{b}_3 \frac{{}^A d b_1}{dt}\cdot\hat{b}_2$$

simple ang vel: ${}^A\bar{\omega}^B = \omega\hat{k}$

$$\frac{{}^A d\bar{v}}{dt} = \frac{{}^B d\bar{v}}{dt} + {}^A\bar{\omega}^B \times \bar{v}$$

Aux. RF

$${}^A\bar{\omega}^B = {}^A\bar{\omega}^{A_1} + {}^{A_1}\bar{\omega}^{A_2} + \ldots + {}^{A_{n-1}}\bar{\omega}^{A_n} + {}^{A_n}\bar{\omega}^B$$

very useful simple rotations

no addition for ${}^A\bar{\alpha}^B$!

Ang acc

$${}^A\bar{\alpha}^B \triangleq \frac{{}^A d\, {}^A\bar{\omega}^B}{dt}$$

simple ang acc

${}^A\bar{\alpha}^B = \alpha\hat{k}$

$$\alpha = \frac{d\omega}{dt}$$

Linear vel & acc

$${}^A\bar{v}^P \triangleq \frac{{}^A d\,\bar{p}}{dt}$$ — position of point

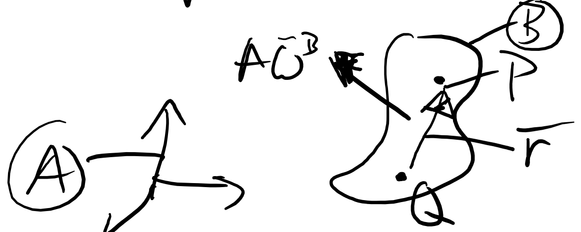
$${}^A\bar{a}^P \triangleq \frac{{}^A d\,{}^A\bar{v}^P}{dt}$$

Two points on rigid body (or RF)

$${}^A\bar{v}^P = {}^A\bar{v}^Q + {}^A\bar{\omega}^B \times \bar{r}^{P/Q}$$

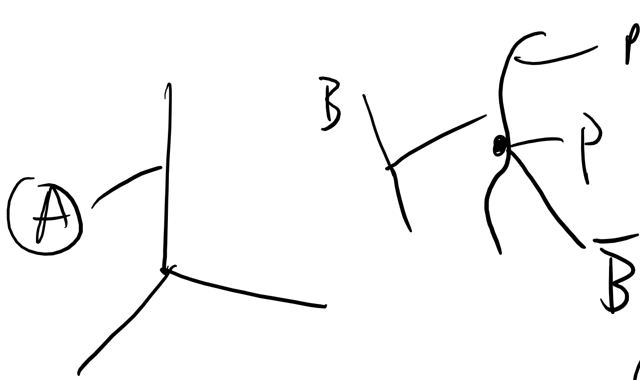
P and Q fixed rel to each other in B

$${}^A\bar{a}^P = {}^A\bar{a}^Q + \overbrace{{}^A\bar{\omega}^B \times \left({}^A\bar{\omega}^B \times \bar{r}^{P/Q}\right)}^{\text{Centripetal acc}} + \underbrace{{}^A\bar{\alpha}^B \times \bar{r}^{P/Q}}_{\text{tangential term}}$$



$\bar{r} \Rightarrow$ constant in B

One points moving on rigid body



$$ {}^A\bar{v}^P = {}^A\bar{v}^{\bar{B}} + {}^B\bar{v}^P, \quad {}^A\bar{a}^P = {}^A\bar{a}^{\bar{B}} + {}^B\bar{a}^P + \overbrace{2\,{}^A\omega^B \times {}^B\bar{v}^P}^{\text{coriolis term}}$$

$\bar{B}$ point fixed in B at that instance of time

$\nu = \#$ particles

Configuration constraints (holonomic)

$n = \min \#$ coordinates (after all config constraints applied)

$$f(x_1, y_1, z_1, \dots, x_\nu, y_\nu, z_\nu, t) = 0$$

$$\underbrace{q_1 = L + q_2}$$

ex. config constraint $\Rightarrow q_1 = f(q_2)$

Motion constraints (nonholonomic)

$$g(x_1, y_1, z_1 \dots x_n, y_n, z_n, \dot{x}_1, \dot{y}_1, \dot{z}_1, \dots, \dot{x}_\nu, \dot{y}_\nu, \dot{z}_\nu, t) = 0$$

if mixed partials commute then it is integrabl and g is not a motion constraint, it is a configuration con.

ex mot. con. $\rightarrow$ ${}^N\bar{V}^P = 0$

$m = \#$ motion constraints

$$\# DoF = p = n - m$$

if g is not integrable (mixed don't commute), it is an essential nonholonomic M.C.